ANALYSIS OF HEAT EXCHANGERS
Overall Heat Transfer Coefficient

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by convection, through the wall by conduction, and from the wall to the cold fluid again by convection. Any radiation effects are usually included in the convection heat transfer coefficients.

For a double-pipe heat exchanger, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$, and the thermal resistance of the tube wall in this case is

$$R_{\text{wall}} = \frac{\ln (D_o/D_i)}{2\pi k L}$$

where $k$ is the thermal conductivity of the wall material and $L$ is the length of the tube. Then the total thermal resistance becomes

$$R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln (D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o}$$
Overall Heat Transfer Coefficient

For tubular heat exchangers we must take into account the conduction resistance in the wall and convection resistances of the fluids at the inner and outer tube surfaces.

\[
\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{\ln(D_o / D_i)}{2\pi k L} + \frac{1}{h_o A_o}
\]

The \( A_i \) is the area of the inner surface of the wall that separates the two fluids, and \( A_o \) is the area of the outer surface of the wall. In other words, \( A_i \) and \( A_o \) are surface areas of the separating wall wetted by the inner and the outer fluids, respectively. When one fluid flows inside a circular tube and the other outside of it, we have:

\[
A_i = \pi D_i L \\
A_o = \pi D_o L
\]

Note that:

\[
\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}
\]

where inner tube surface

where outer tube surface
Overall Heat Transfer Coefficient

In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance \( R \), and to express the rate of heat transfer between the two fluids as

\[
\dot{Q} = \frac{\Delta T}{R} = UA \Delta T = U_i A_i \Delta T = U_o A_o \Delta T
\]

where \( U \) is the overall heat transfer coefficient, whose unit is W/m\(^2\) \cdot °C, which is identical to the unit of the ordinary convection coefficient \( h \). Canceling \( \Delta T \), Eq. reduces to

\[
\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{\text{wall}} + \frac{1}{h_o A_o}
\]

Perhaps you are wondering why we have two overall heat transfer coefficients \( U_i \) and \( U_o \) for a heat exchanger. The reason is that every heat exchanger has two heat transfer surface areas \( A_i \) and \( A_o \), which, in general, are not equal to each other.

Then Eq. for the overall heat transfer coefficient simplifies to

\[
\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}
\]

where \( U \approx U_i \approx U_o \). \( h_i \) and \( h_o \) are the individual convection heat transfer coefficients inside and outside the tube,
# Overall Heat Transfer Coefficient

<table>
<thead>
<tr>
<th>Type of heat exchanger</th>
<th>$U$, W/m² · °C*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water-to-water</td>
<td>850–1700</td>
</tr>
<tr>
<td>Water-to-oil</td>
<td>100–350</td>
</tr>
<tr>
<td>Water-to-gasoline or kerosene</td>
<td>300–1000</td>
</tr>
<tr>
<td>Feedwater heaters</td>
<td>1000–8500</td>
</tr>
<tr>
<td>Steam-to-light fuel oil</td>
<td>200–400</td>
</tr>
<tr>
<td>Steam-to-heavy fuel oil</td>
<td>50–200</td>
</tr>
<tr>
<td>Steam condenser</td>
<td>1000–6000</td>
</tr>
<tr>
<td>Freon condenser (water cooled)</td>
<td>300–1000</td>
</tr>
<tr>
<td>Ammonia condenser (water cooled)</td>
<td>800–1400</td>
</tr>
<tr>
<td>Alcohol condensers (water cooled)</td>
<td>250–700</td>
</tr>
<tr>
<td>Gas-to-gas</td>
<td>10–40</td>
</tr>
<tr>
<td>Water-to-air in finned tubes (water in tubes)</td>
<td>30–60†</td>
</tr>
<tr>
<td></td>
<td>400–850†</td>
</tr>
<tr>
<td>Steam-to-air in finned tubes (steam in tubes)</td>
<td>30–300†</td>
</tr>
</tbody>
</table>
|                                              | 400–4000‡
Fouling

Accumulation of undesirable deposit on a HEX surface.
The performance of heat exchangers usually deteriorates with time as a result of accumulation of deposits on heat transfer surfaces. The layer of deposits represents additional resistance to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease.

The net effect of these accumulations on heat transfer is represented by a **Fouling factor**, $R_f$.

- The overall heat transfer coefficient can be written:

\[
\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln(D_o / D_i)}{2\pi k L} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o}
\]

where $A_i = \pi D_i L$ and $A_o = \pi D_o L$ are the areas of inner and outer surfaces, and $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.
Fouling

Some values of fouling factors are given here. More comprehensive tables of fouling factors are available in handbooks. As you would expect, considerable uncertainty exists in these values, and they should be used as a guide in the selection and evaluation of heat exchangers to account for the effects of anticipated fouling on heat transfer.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$R_f$, m²·°C/W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distilled water, seawater, river water, boiler feedwater: Below 50°C</td>
<td>0.0001</td>
</tr>
<tr>
<td>Above 50°C</td>
<td>0.0002</td>
</tr>
<tr>
<td>Fuel oil</td>
<td>0.0009</td>
</tr>
<tr>
<td>Steam (oil-free)</td>
<td>0.0001</td>
</tr>
<tr>
<td>Refrigerants (liquid)</td>
<td>0.0002</td>
</tr>
<tr>
<td>Refrigerants (vapor)</td>
<td>0.0004</td>
</tr>
<tr>
<td>Alcohol vapors</td>
<td>0.0001</td>
</tr>
<tr>
<td>Air</td>
<td>0.0004</td>
</tr>
</tbody>
</table>
EXAMPLE

Overall Heat Transfer Coefficient of a Heat Exchanger

Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tubes have a diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (the shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger.

SOLUTION Hot oil is cooled by water in a double-tube counter-flow heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions 1 The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. 2 Both the oil and water flow are fully developed. 3 Properties of the oil and water are constant.

Properties The properties of water at 45°C are (Table A–15)

\[ \rho = 990 \text{ kg/m}^3 \quad \text{Pr} = 3.91 \]
\[ k = 0.637 \text{ W/m} \cdot ^\circ \text{C} \quad \nu = \mu/\rho = 0.602 \times 10^{-6} \text{ m}^2/\text{s} \]
The properties of oil at 80°C are (Table A–19).

\[ \rho = 852 \text{ kg/m}^3 \quad \text{Pr} = 490 \]

\[ k = 0.138 \text{ W/m} \cdot ^\circ\text{C} \quad v = 37.5 \times 10^{-6} \text{ m}^2/\text{s} \]

**Analysis**  The schematic of the heat exchanger is given in Fig. 23–10. The overall heat transfer coefficient \( U \) can be determined from Eq. 23–5:

\[ \frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o} \]

where \( h_i \) and \( h_o \) are the convection heat transfer coefficients inside and outside the tube, respectively, which are to be determined using the forced convection relations.

The hydraulic diameter for a circular tube is the diameter of the tube itself, \( D_h = D = 0.02 \text{ m} \). The mean velocity of water in the tube and the Reynolds number are

\[ \nu_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho (\frac{1}{4} \pi D^2)} = \frac{0.5 \text{ kg/s}}{(990 \text{ kg/m}^3)[\frac{1}{4} \pi (0.02 \text{ m})^2]} = 1.61 \text{ m/s} \]

and

\[ \text{Re} = \frac{\nu_m D_h}{v} = \frac{(1.61 \text{ m/s})(0.02 \text{ m})}{0.602 \times 10^{-6} \text{ m}^2/\text{s}} = 53,490 \]

which is greater than 4000. Therefore, the flow of water is turbulent. Assuming the flow to be fully developed, the Nusselt number can be determined from

\[ \text{Nu} = \frac{hD_h}{k} = 0.023 \text{ Re}^{0.8} \text{Pr}^{0.4} = 0.023(53,490)^{0.8}(3.91)^{0.4} = 240.6 \]
Then, \[ h = \frac{k}{D_h} \text{Nu} = \frac{0.637 \text{ W/m \cdot °C}}{0.02 \text{ m}} (240.6) = 7663 \text{ W/m}^2 \cdot \text{°C} \]

Now we repeat the analysis above for oil. The properties of oil at 80°C are
\[
\rho = 852 \text{ kg/m}^3 \quad \nu = 37.5 \times 10^{-6} \text{ m}^2/\text{s} \\
k = 0.138 \text{ W/m \cdot °C} \quad \text{Pr} = 490
\]

The hydraulic diameter for the annular space is
\[ D_h = D_o - D_i = 0.03 - 0.02 = 0.01 \text{ m} \]

The mean velocity and the Reynolds number in this case are
\[
\dot{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho \left[ \frac{1}{4} \pi (D_o^2 - D_i^2) \right]} = \frac{0.8 \text{ kg/s}}{(852 \text{ kg/m}^3)[\frac{1}{4} \pi (0.03^2 - 0.02^2)] \text{ m}^2} = 2.39 \text{ m/s}
\]

and
\[ \text{Re} = \frac{\dot{V}_m D_h}{\nu} = \frac{(2.39 \text{ m/s})(0.01 \text{ m})}{37.5 \times 10^{-6} \text{ m}^2/\text{s}} = 637 \]

which is less than 4000. Therefore, the flow of oil is laminar. Assuming fully developed flow, the Nusselt number on the tube side of the annular space \( \text{Nu}_i \) corresponding to \( D_i/D_o = 0.02/0.03 = 0.667 \) can be determined from Table 23–3 by interpolation to be \( \text{Nu} = 5.45 \)

and
\[ h_o = \frac{k}{D_h} \text{Nu} = \frac{0.138 \text{ W/m \cdot °C}}{0.01 \text{ m}} (5.45) = 75.2 \text{ W/m}^2 \cdot \text{°C} \]

Then the overall heat transfer coefficient for this heat exchanger becomes
\[
U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{7663 \text{ W/m}^2 \cdot \text{°C}} + \frac{1}{75.2 \text{ W/m}^2 \cdot \text{°C}}} = 74.5 \text{ W/m}^2 \cdot \text{°C}
\]

<table>
<thead>
<tr>
<th>( D_i/D_o )</th>
<th>( \text{Nu}_i )</th>
<th>( \text{Nu}_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>—</td>
<td>3.66</td>
</tr>
<tr>
<td>0.05</td>
<td>17.46</td>
<td>4.06</td>
</tr>
<tr>
<td>0.10</td>
<td>11.56</td>
<td>4.11</td>
</tr>
<tr>
<td>0.25</td>
<td>7.37</td>
<td>4.23</td>
</tr>
<tr>
<td>0.50</td>
<td>5.74</td>
<td>4.43</td>
</tr>
<tr>
<td>1.00</td>
<td>4.86</td>
<td>4.86</td>
</tr>
</tbody>
</table>
EXAMPLE 23–2  Effect of Fouling on the Overall Heat Transfer Coefficient

A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel \( k = 15.1 \text{ W/m} \cdot \text{°C} \) inner tube of inner diameter \( D_i = 1.5 \text{ cm} \) and outer diameter \( D_o = 1.9 \text{ cm} \) and an outer shell of inner diameter 3.2 cm. The convection heat transfer coefficient is given to be \( h_i = 800 \text{ W/m}^2 \cdot \text{°C} \) on the inner surface of the tube and \( h_o = 1200 \text{ W/m}^2 \cdot \text{°C} \) on the outer surface. For a fouling factor of \( R_{f,i} = 0.0004 \text{ m}^2 \cdot \text{°C/W} \) on the tube side and \( R_{f,o} = 0.0001 \text{ m}^2 \cdot \text{°C/W} \) on the shell side, determine (a) the thermal resistance of the heat exchanger per unit length and (b) the overall heat transfer coefficients, \( U_i \) and \( U_o \) based on the inner and outer surface areas of the tube, respectively.

SOLUTION  The heat transfer coefficients and the fouling factors on the tube and shell sides of a heat exchanger are given. The thermal resistance and the overall heat transfer coefficients based on the inner and outer areas are to be determined.

Assumptions  The heat transfer coefficients and the fouling factors are constant and uniform.

Analysis  (a) The schematic of the heat exchanger is given in Fig. 23–11. The thermal resistance for an unfinned shell-and-tube heat exchanger with fouling on both heat transfer surfaces is given by Eq. 23–8 as

\[
R = \frac{1}{UA_x} = \frac{1}{U_iA_i} + \frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{1}{h_oA_o} + \frac{R_{f,i}}{A_i} + \frac{R_{f,o}}{A_o} + \frac{\ln \left( \frac{D_o}{D_i} \right)}{2\pi kL} + \frac{1}{h_oA_o}
\]

where

\[
A_i = \pi D_i L = \pi(0.015 \text{ m})(1 \text{ m}) = 0.0471 \text{ m}^2
\]

\[
A_o = \pi D_o L = \pi(0.019 \text{ m})(1 \text{ m}) = 0.0597 \text{ m}^2
\]

Substituting, the total thermal resistance is determined to be
\[ R = \frac{1}{(800 \text{ W/m}^2 \cdot ^\circ \text{C})(0.0471 \text{ m}^2)} + \frac{0.0004 \text{ m}^2 \cdot ^\circ \text{C}/\text{W}}{0.0471 \text{ m}^2} \]
\[ + \frac{\ln (0.019/0.015)}{2\pi(15.1 \text{ W/m} \cdot ^\circ \text{C})(1 \text{ m})} \]
\[ + \frac{0.0001 \text{ m}^2 \cdot ^\circ \text{C}/\text{W}}{0.0597 \text{ m}^2} + \frac{1}{(1200 \text{ W/m}^2 \cdot ^\circ \text{C})(0.0597 \text{ m}^2)} \]
\[ = (0.02654 + 0.00849 + 0.0025 + 0.00168 + 0.01396)^\circ \text{C}/\text{W} \]
\[ = 0.0532^\circ \text{C}/\text{W} \]

Note that about 19 percent of the total thermal resistance in this case is due to fouling and about 5 percent of it is due to the steel tube separating the two fluids. The rest (76 percent) is due to the convection resistances on the two sides of the inner tube.

(b) Knowing the total thermal resistance and the heat transfer surface areas, the overall heat transfer coefficients based on the inner and outer surfaces of the tube are determined again from Eq. 23–8 to be

\[ U_i = \frac{1}{RA_i} = \frac{1}{(0.0532 ^\circ \text{C}/\text{W})(0.0471 \text{ m}^2)} = 399 \text{ W/m}^2 \cdot ^\circ \text{C} \]

and

\[ U_o = \frac{1}{RA_o} = \frac{1}{(0.0532 ^\circ \text{C}/\text{W})(0.0597 \text{ m}^2)} = 315 \text{ W/m}^2 \cdot ^\circ \text{C} \]
The log mean temperature difference $\Delta T_{lm}$ relation developed earlier is limited to parallel-flow and counter-flow heat exchangers only. In such cases, it is convenient to relate the equivalent temperature difference to the log mean temperature difference relation for the counter-flow case as:

$$\Delta T_{lm} = F \Delta T_{lm,CF}$$

where $F$ is the correction factor, which depends on the geometry of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams. The $\Delta T_{lm,CF}$ is the log mean temperature difference for the case of a counter-flow heat exchanger with the same inlet and outlet temperatures and is determined from the above equation, and by taking:

$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$ and $\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$ as shown in the figure.
The correction factor is less than unity for a cross-flow and multipass shell-and-tube heat exchanger. That is, \( F \leq 1 \). The limiting value of \( F = 1 \) corresponds to the counter-flow heat exchanger. Thus, the correction factor \( F \) for a heat exchanger is a measure of deviation of the \( \Delta T_{im} \) from the corresponding values for the counter-flow case.

The correction factor \( F \) for common cross-flow and shell-and-tube heat exchanger configurations is given in Fig. 23–18 versus two temperature ratios \( P \) and \( R \) defined as

\[
P = \frac{t_2 - t_1}{T_1 - t_1} \quad (23–27)
\]

and

\[
R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(mc_p)_{\text{tube side}}}{(mc_p)_{\text{shell side}}} \quad (23–28)
\]

where the subscripts 1 and 2 represent the inlet and outlet, respectively.

Note that for a shell-and-tube heat exchanger, \( T \) and \( t \) represent the shell- and tube-side temperatures, respectively, as shown in the correction factor charts.
Correction Factor

- Correction factor parameters, \( R \) and \( P \)
  - Shell and tube definitions below

\[
P = \frac{T_{\text{tube, out}} - T_{\text{tube, in}}}{T_{\text{shell, in}} - T_{\text{tube, in}}} = \frac{t_2 - t_1}{T_1 - t_1}
\]

\[
R = \frac{T_{\text{shell, in}} - T_{\text{tube, in}}}{T_{\text{tube, out}} - T_{\text{tube, in}}} = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(mcP)_{\text{tube}}}{(mcP)_{\text{shell}}}
\]

- Correction factor charts show diagrams that illustrate the equations for \( P \) and \( R \)
Example (3) : Cross Flow Heat exchangers

Cross flow heat exchanger is used to heat an oil in the tubes ($c = 1.9 \text{ kJ/kg \cdot °C}$) from 15°C to 85°C. Blowing across the outside of the tubes is steam which enters at 130°C and leaves at 110°C with a mass-flow of 5.2 kg/sec. The overall heat-transfer coefficient is 275 W/m² \cdot °C and $c$ for steam is 1.86 kJ/kg \cdot °C. Calculate the surface area of the heat exchanger.

Solution

The total heat transfer may be obtained from an energy balance on the steam

$$q = \dot{m}_s c_s \Delta T_s = (5.2)(1.86)(130 - 110) = 193 \text{ kW}$$
\[ \Delta T_m = \frac{(130 - 85) - (110 - 15)}{\ln \left( \frac{130 - 85}{110 - 15} \right)} = 66.9^\circ C \]

Now, from Fig. 10-11, \( t_1 \) and \( t_2 \) will represent the unmixed fluid (the oil) and \( T_1 \) and \( T_2 \) will represent the mixed fluid (the steam) so that

\[ T_1 = 130 \quad T_2 = 110 \quad t_1 = 15 \quad t_2 = 85^\circ C \]

and we calculate

\[ R = \frac{130 - 110}{85 - 15} = 0.286 \quad P = \frac{85 - 15}{130 - 15} = 0.609 \]

Consulting Fig. 10-11 we find

\[ F = 0.97 \]

so the area is calculated from

\[ A = \frac{q}{UF \Delta T_m} = \frac{193,000}{(275)(0.97)(66.9)} = 10.82 \text{ m}^2 \]
Example (4)

Investigate the heat-transfer performance of the HEX in example 3 if the oil flow rate is reduced in half while the steam flow remains same. Assume $U$ remains constant at $275 \ W/m^2.°C$

Solution

Using,

$$q = mc_p \Delta T$$

$$\dot{m}_o = \frac{193}{(1.9)(85 - 15)} = 1.45 \ kg/s$$

The new flow rate will be half this value or $0.725 \ kg/s$. We are assuming the inlet temperatures remain the same at $130°C$ for the steam and $15°C$ for the oil. The new relation for the heat transfer is
\[ Q = mC_p(t_2 - 15) = mC_p(130 - T_1) \]  \hspace{1cm} (a)

\[ Q = U A T_{LMTD} F \]  \hspace{1cm} (b)

The general procedure is to assume values of the exit temperatures until the \( q \)'s agree between Eqs. (a) and (b).

This why there was a need for another easier method

Effective-Ntu method
Multipass and Cross-Flow Heat Exchangers

(a) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes

(b) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes
Multipass and Cross-Flow Heat Exchangers

**FIGURE 23–18**
Correction factor F charts for common shell-and-tube and cross-flow heat exchangers
(d) Single-pass cross-flow with one fluid \textit{mixed} and the other \textit{unmixed}
EXAMPLE  The Condensation of Steam in a Condenser

Steam in the condenser of a power plant is to be condensed at a temperature of 30°C with cooling water from a nearby lake, which enters the tubes of the condenser at 14°C and leaves at 22°C. The surface area of the tubes is 45 m², and the overall heat transfer coefficient is 2100 W/m² · °C. Determine the mass flow rate of the cooling water needed and the rate of condensation of the steam in the condenser.

SOLUTION  Steam is condensed by cooling water in the condenser of a power plant. The mass flow rate of the cooling water and the rate of condensation are to be determined.

Assumptions  1 Steady operating conditions exist. 2 The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. 3 Changes in the kinetic and potential energies of fluid streams are negligible. 4 There is no fouling. 5 Fluid properties are constant.

Properties  The heat of vaporization of water at 30°C is \( h_{fg} = 2431 \text{ kJ/kg} \) and the specific heat of cold water at the average temperature of 18°C is \( C_p = 4184 \text{ J/kg · °C} \) (Table A–15).

Analysis  The schematic of the condenser is given in Fig. 23–19. The condenser can be treated as a counter-flow heat exchanger since the temperature of one of the fluids (the steam) remains constant.

The temperature difference between the steam and the cooling water at the two ends of the condenser is

\[
\Delta T_1 = T_{h, \text{in}} - T_{c, \text{out}} = (30 - 22)\text{°C} = 8\text{°C} \\
\Delta T_2 = T_{h, \text{out}} - T_{c, \text{in}} = (30 - 14)\text{°C} = 16\text{°C}
\]
That is, the temperature difference between the two fluids varies from 8°C at one end to 16°C at the other. The proper average temperature difference between the two fluids is the logarithmic mean temperature difference (not the arithmetic), which is determined from

\[
\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{8 - 16}{\ln(8/16)} = 11.5^\circ C
\]

This is a little less than the arithmetic mean temperature difference of \(\frac{1}{2}(8 + 16) = 12^\circ C\). Then the heat transfer rate in the condenser is determined from

\[
\dot{Q} = U A \Delta T_{\text{lm}} = (2100 \text{ W/m}^2 \cdot ^\circ C)(45 \text{ m}^2)(11.5^\circ C) = 1.087 \times 10^6 \text{ W} = 1087 \text{ kW}
\]

Therefore, the steam will lose heat at a rate of 1,087 kW as it flows through the condenser, and the cooling water will gain practically all of it, since the condenser is well insulated.

The mass flow rate of the cooling water and the rate of the condensation of the steam are determined from \(\dot{Q} = [\dot{m} C_p (T_{\text{out}} - T_{\text{in}})]_{\text{cooling water}} = (\dot{m} h_{fg})_{\text{steam}}\) to be

\[
\dot{m}_{\text{cooling water}} = \frac{\dot{Q}}{C_p(T_{\text{out}} - T_{\text{in}})} = \frac{1087 \text{ kJ/s}}{(4.184 \text{ kJ/kg} \cdot ^\circ C)(22 - 14)^\circ C} = 32.5 \text{ kg/s}
\]

and

\[
\dot{m}_{\text{steam}} = \frac{\dot{Q}}{h_{fg}} = \frac{1087 \text{ kJ/s}}{2431 \text{ kJ/kg}} = 0.45 \text{ kg/s}
\]

Therefore, we need to circulate about 72 kg of cooling water for each 1 kg of steam condensing to remove the heat released during the condensation process.
EXAMPLE 23–4  Heating Water in a Counter-Flow Heat Exchanger

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m²·°C, determine the length of the heat exchanger required to achieve the desired heating.

SOLUTION  Water is heated in a counter-flow double-pipe heat exchanger by geothermal water. The required length of the heat exchanger is to be determined.

Assumptions  1 Steady operating conditions exist. 2 The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. 3 Changes in the kinetic and potential energies of fluid streams are negligible. 4 There is no fouling. 5 Fluid properties are constant.

Properties  We take the specific heats of water and geothermal fluid to be 4.18 and 4.31 kJ/kg·°C, respectively.

Analysis  The schematic of the heat exchanger is given in Fig. 23–20. The rate of heat transfer in the heat exchanger can be determined from

\[ \dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{water} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot ^\circ \text{C})(80 - 20)^\circ \text{C} = 301 \text{ kW} \]

Noting that all of this heat is supplied by the geothermal water, the outlet temperature of the geothermal water is determined to be

\[ \dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{geothermal} \implies T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p} \]

\[ = 160^\circ \text{C} - \frac{301 \text{ kW}}{(2 \text{ kg/s})(4.31 \text{ kJ/kg} \cdot ^\circ \text{C})} \]

\[ = 125^\circ \text{C} \]
Knowing the inlet and outlet temperatures of both fluids, the logarithmic mean temperature difference for this counter-flow heat exchanger becomes

\[ \Delta T_1 = T_{h, \text{in}} - T_{c, \text{out}} = (160 - 80)\degree C = 80\degree C \]
\[ \Delta T_2 = T_{h, \text{out}} - T_{c, \text{in}} = (125 - 20)\degree C = 105\degree C \]

and

\[ \Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{80 - 105}{\ln(80/105)} = 92.0\degree C \]

Then the surface area of the heat exchanger is determined to be

\[ \dot{Q} = UA_s \Delta T_{\text{lm}} \quad \Rightarrow \quad A_s = \frac{\dot{Q}}{U \Delta T_{\text{lm}}} = \frac{301,000 \text{ W}}{(640 \text{ W/m}^2 \cdot \degree C)(92.0\degree C)} = 5.11 \text{ m}^2 \]

To provide this much heat transfer surface area, the length of the tube must be

\[ A_s = \pi DL \quad \Rightarrow \quad L = \frac{A_s}{\pi D} = \frac{5.11 \text{ m}^2}{\pi(0.015 \text{ m})} = 108 \text{ m} \]

**Discussion**  The inner tube of this counter-flow heat exchanger (and thus the heat exchanger itself) needs to be over 100 m long to achieve the desired heat transfer, which is impractical. In cases like this, we need to use a plate heat exchanger or a multipass shell-and-tube heat exchanger with multiple passes of tube bundles.
EXAMPLE 23-5  Heating of Glycerin in a Multipass Heat Exchanger

A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin-walled 2-cm-diameter tubes at 80°C and leaves at 40°C (Fig. 23–21). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m² · °C on the glycerin (shell) side and 160 W/m² · °C on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (a) before any fouling occurs and (b) after fouling with a fouling factor of 0.0006 m² · °C/W occurs on the outer surfaces of the tubes.

SOLUTION  Glycerin is heated in a 2-shell passes and 4-tube passes heat exchanger by hot water. The rate of heat transfer for the cases of fouling and no fouling are to be determined.

Assumptions  1 Steady operating conditions exist. 2 The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. 3 Changes in the kinetic and potential energies of fluid streams are negligible. 4 Heat transfer coefficients and fouling factors are constant and uniform. 5 The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Analysis  The tubes are said to be thin-walled, and thus it is reasonable to assume the inner and outer surface areas of the tubes to be equal. Then the heat transfer surface area becomes

\[ A_s = \pi DL = \pi (0.02 \text{ m})(60 \text{ m}) = 3.77 \text{ m}^2 \]

The rate of heat transfer in this heat exchanger can be determined from

\[ \dot{Q} = UA_s F \Delta T_{\text{m,CF}} \]
The rate of heat transfer in this heat exchanger can be determined from

\[ \dot{Q} = UA_r F \Delta T_{\text{lm, CF}} \]

where \( F \) is the correction factor and \( \Delta T_{\text{lm, CF}} \) is the log mean temperature difference for the counter-flow arrangement. These two quantities are determined from

\[ \Delta T_1 = T_{h, \text{in}} - T_{c, \text{out}} = (80 - 50)\degree \text{C} = 30\degree \text{C} \]
\[ \Delta T_2 = T_{h, \text{out}} - T_{c, \text{in}} = (40 - 20)\degree \text{C} = 20\degree \text{C} \]
\[ \Delta T_1 - \Delta T_2 = 30 - 20 \]

and

\[ P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{40 - 80}{20 - 80} = 0.67 \]
\[ F = 0.91 \quad \text{(Fig. 23–18b)} \]

\[ R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 50}{40 - 80} = 0.75 \]

(a) In the case of no fouling, the overall heat transfer coefficient \( U \) is determined from

\[ U = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{160 \, \text{W/m}^2 \cdot \degree \text{C}} + \frac{1}{25 \, \text{W/m}^2 \cdot \degree \text{C}} = 21.6 \, \text{W/m}^2 \cdot \degree \text{C} \]

Then the rate of heat transfer becomes

\[ \dot{Q} = UA_r F \Delta T_{\text{lm, CF}} = (21.6 \, \text{W/m}^2 \cdot \degree \text{C})(3.77 \, \text{m}^2)(0.91)(24.7\degree \text{C}) = 1830 \, \text{W} \]
(b) When there is fouling on one of the surfaces, the overall heat transfer coefficient $U$ is

$$U = \frac{1}{h_i} + \frac{1}{h_o} + R_f \left( \frac{1}{160 \text{ W/m}^2 \cdot ^\circ \text{C}} + \frac{1}{25 \text{ W/m}^2 \cdot ^\circ \text{C}} + 0.0006 \text{ m}^2 \cdot ^\circ \text{C}/\text{W} \right)$$

$$= 21.3 \text{ W/m}^2 \cdot ^\circ \text{C}$$

The rate of heat transfer in this case becomes

$$\dot{Q} = UA_s F \Delta T_{lm,CF} = (21.3 \text{ W/m}^2 \cdot ^\circ \text{C})(3.77 \text{ m}^2)(0.91)(24.7^\circ \text{C}) = \boxed{1805 \text{ W}}$$

**Discussion**  Note that the rate of heat transfer decreases as a result of fouling, as expected. The decrease is not dramatic, however, because of the relatively low convection heat transfer coefficients involved.
EXAMPLE 23–6  Cooling of an Automotive Radiator

A test is conducted to determine the overall heat transfer coefficient in an automotive radiator that is a compact cross-flow water-to-air heat exchanger with both fluids (air and water) unmixed (Fig. 23–22). The radiator has 40 tubes of internal diameter 0.5 cm and length 65 cm in a closely spaced plate-finned matrix. Hot water enters the tubes at 90°C at a rate of 0.6 kg/s and leaves at 65°C. Air flows across the radiator through the interfin spaces and is heated from 20°C to 40°C. Determine the overall heat transfer coefficient \( U \) of this radiator based on the inner surface area of the tubes.

**SOLUTION**  During an experiment involving an automotive radiator, the inlet and exit temperatures of water and air and the mass flow rate of water are measured. The overall heat transfer coefficient based on the inner surface area is to be determined.

**Assumptions**  1 Steady operating conditions exist. 2 Changes in the kinetic and potential energies of fluid streams are negligible. 3 Fluid properties are constant.

**Properties**  The specific heat of water at the average temperature of \((90 + 65)/2 = 77.5°C\) is 4.195 kJ/kg \( \cdot \) °C.

**Analysis**  The rate of heat transfer in this radiator from the hot water to the air is determined from an energy balance on water flow,

\[
\dot{Q} = [\dot{m} C_p (T_{in} - T_{out})]_{water} = (0.6 \text{ kg/s})(4.195 \text{ kJ/kg } \cdot \text{ °C})(90 - 65)\text{°C} = 62.93 \text{ kW}
\]

The tube-side heat transfer area is the total surface area of the tubes, and is determined from

\[
A_t = n \pi D_t L = (40)\pi(0.005 \text{ m})(0.65 \text{ m}) = 0.408 \text{ m}^2
\]
Knowing the rate of heat transfer and the surface area, the overall heat transfer coefficient can be determined from

\[ \dot{Q} = U_i A_i F \Delta T_{\text{lm, CF}} \quad \iff \quad U_i = \frac{\dot{Q}}{A_i F \Delta T_{\text{lm, CF}}} \]

where \( F \) is the correction factor and \( \Delta T_{\text{lm, CF}} \) is the log mean temperature difference for the counter-flow arrangement. These two quantities are found to be

\[ \Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}} = (90 - 40) \, ^{\circ}\!\!C = 50 \, ^{\circ}\!\!C \]
\[ \Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}} = (65 - 20) \, ^{\circ}\!\!C = 45 \, ^{\circ}\!\!C \]
\[ \Delta T_{\text{lm, CF}} = \frac{\Delta T_1 - \Delta T_2}{\ln \left( \frac{\Delta T_1}{\Delta T_2} \right)} = \frac{50 - 45}{\ln (50/45)} = 47.6 \, ^{\circ}\!\!C \]

and

\[
\begin{align*}
P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{65 - 90}{20 - 90} = 0.36 \\
R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 40}{65 - 90} = 0.80
\end{align*}
\]

\[ F = 0.97 \quad (\text{Fig. 23–18c}) \]

Substituting, the overall heat transfer coefficient \( U_i \) is determined to be

\[ U_i = \frac{\dot{Q}}{A_i F \Delta T_{\text{lm, CF}}} = \frac{62,930 \, \text{W}}{(0.408 \, \text{m}^2)(0.97)(47.6 \, ^{\circ}\!\!C)} = 3341 \, \text{W/m}^2 \cdot ^{\circ}\!\!C \]

Note that the overall heat transfer coefficient on the air side will be much lower because of the large surface area involved on that side.
THE EFFECTIVENESS–NTU METHOD

Once $\Delta T_{lm}$, the mass flow rates, and the overall heat transfer coefficient ($U$) are available, the heat transfer surface area of the heat exchanger can be determined from

$$\dot{Q} = UA_s \Delta T_{lm}$$

Therefore, the LMTD method is very suitable for determining the size of a heat exchanger to realize prescribed outlet temperatures when the mass flow rates and the inlet and outlet temperatures of the hot and cold fluids are specified.

With the LMTD method, the task is to select a heat exchanger that will meet the prescribed heat transfer requirements. The procedure to be followed by the selection process is:

1. Select the type of heat exchanger suitable for the application.
2. Determine any unknown inlet or outlet temperature and the heat transfer rate using an energy balance.
3. Calculate the log mean temperature difference $\Delta T_{lm}$ and the correction factor $F$, if necessary.
4. Obtain (select or calculate) the value of the overall heat transfer coefficient $U$.
5. Calculate the heat transfer surface area $A_s$.
This method is based on a dimensionless parameter called the heat transfer effectiveness $\varepsilon$, defined as:

$$\varepsilon = \frac{\dot{Q}}{Q_{\text{max}}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

The actual heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as:

$$\dot{Q} = C_c(T_{c,\text{out}} - T_{c,\text{in}}) = C_h(T_{h,\text{in}} - T_{h,\text{out}})$$

To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the maximum temperature difference:

$$\Delta T_{\text{max}} = T_{h,\text{in}} - T_{c,\text{in}}$$

the maximum possible heat transfer rate in a heat exchanger is:

$$\dot{Q}_{\text{max}} = C_{\min} \Delta T_{\text{max}} = 10,120 \text{ kW}$$

where $C_{\min}$ is the smaller of $C_h = \dot{m}_h C_{ph}$ and $C_c = m_c C_{pc}$.
Consider two counter-flow heat exchangers, one in which the cold fluid has the larger \( \Delta T \) (smaller \( m \cdot c_p \)) and a second in which the cold fluid has the smaller \( \Delta T \) (larger \( m \cdot c_p \)):

The effectiveness is the ratio of the energy recovered in a HX to that recoverable in an ideal HX.

\[
\varepsilon = \frac{\dot{m} \cdot c_p \cdot (t_2 - t_1)}{\dot{m} \cdot c_p \cdot (T_1 - t_1)} \quad \Delta t > \Delta T
\]

\[
\varepsilon = \frac{\dot{M} \cdot C_p \cdot (T_1 - T_2)}{\dot{M} \cdot C_p \cdot (T_1 - t_1)} \quad \Delta T > \Delta t
\]

Canceling identical terms from the numerator and denominator of both terms:

\[
\varepsilon = \frac{(t_2 - t_1)}{(T_1 - t_1)} \quad \Delta t > \Delta T
\]

\[
\varepsilon = \frac{(T_1 - T_2)}{(T_1 - t_1)} \quad \Delta T > \Delta t
\]

We see that the numerator, in the two cases, is the temperature change for the stream having the larger temperature change. The denominator is the same in either case:

\[
\varepsilon = \frac{\Delta T_{\text{max}}}{(T_1 - t_1)}
\]

In the LMTD-F_t method an effectiveness was defined:

\[
P = \frac{t_2 - t_1}{T_1 - t_1}
\]

Note that the use of the upper case \( T \) in the numerator, in contrast to our normal terminology, does not indicate that the hot fluid temperature change is used here.
Example: Double pipe heat exchanger

Water at the rate of 68 kg/min is heated from 35 to 75 °C by an oil having a specific heat of 1.9 kJ/kg. °C. The fluids are used in counterflow double pipe heat exchanger, and the oil enters the exchanger at 110 °C and leaves at 75 °C. The overall heat-transfer coefficient is 320 W/m². °C. Calculate the heat exchanger area.

Solution: The total heat transfer is determined from the energy absorbed by the water.

\[ q = \dot{m}_w c_w \Delta T_w = (68)(4180)(75-35) = 11.37 \text{ MJ/min} \]
\[ = 189.5 \text{ kW} \]

Since all the fluid temperatures are known, the LMTD can be calculated by using the temperature scheme.

\[ \Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)} \]

\[ \Delta T_{lm} = \frac{(110 - 75)(75 - 35)}{\ln(110 - 75)/(75 - 35)} = 37.44 \text{ °C} \]

From the expression: \[ q = UA\Delta T_m \]

\[ A = \frac{1.895 \times 10^5}{(320)(37.44)} = 15.82 \text{ m}^2 = 170 \text{ ft}^2 \]
Continue Example (1) : (1:2) Shell & Tube heat exchanger, knowing that water is in the shell side and oil is in the tube side, recalculate the needed area ..??

\[
P = \frac{\Delta T_{\text{out}}}{\Delta T_{\text{in}}} = \frac{t_2 - t_1}{T_1 - t_1}
\]

\[
R = \frac{\Delta T_{\text{in}}}{\Delta T_{\text{out}}} = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(mc_p)_{\text{tube}}}{(mc_p)_{\text{shell}}}
\]

so the correction factor is

\[F = 0.81\]

and the heat transfer is

\[q = UAF \Delta T_m\]

so that

\[A = \frac{1.895 \times 10^4}{(320)(0.81)(37.44)} = 19.53 \text{ m}^2 \quad [210 \text{ ft}^2]\]
A thin-walled concentric tube heat exchanger of 0.19-m length is to be used to heat deionized water from 40 to 60°C at a flow rate of 5 kg/s. The deionized water flows through the inner tube of 30-mm diameter while hot process water at 95°C flows in the annulus formed with the outer tube of 60-mm diameter. The thermo physical properties of the fluids are:

<table>
<thead>
<tr>
<th></th>
<th>DEIONIZED WATER</th>
<th>PROCESS WATER</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ (kg/m³)</td>
<td>982.3</td>
<td>967.1</td>
</tr>
<tr>
<td>c_p (J/kg.K)</td>
<td>4181</td>
<td>4197</td>
</tr>
<tr>
<td>k (W/m.K)</td>
<td>0.643</td>
<td>0.673</td>
</tr>
<tr>
<td>μ (N.s/m²)*10⁶</td>
<td>548</td>
<td>324</td>
</tr>
<tr>
<td>pr</td>
<td>3.56</td>
<td>2.02</td>
</tr>
</tbody>
</table>

Find:
(1) minimum flow rate required for the hot process water, (b) required overall heat transfer coefficient and whether it is possible to accomplish this heating, and (c) for CF arrangements minimum process water flow required and the effectiveness?
Assumptions:
1. Negligible heat loss to surroundings.
2. Negligible kinetic and potential energy changes.

Analysis: (a) from overall energy balances,

\[ q = (mc)_h(T_{h,i} - T_{h,o}) = (mc)_h(T_{c,o} - T_{c,i}) \]

For a fixed term \( T_{h,i} \), \((m)_h\) will be a minimum when \( T_{h,o} \) is a minimum. With the parallel flow configuration, this requires that \( T_{h,o} = T_{c,o} = 60^\circ C \). Hence,

\[ m_{h, \text{min}} = \frac{(mc)_c(T_{c,o} - T_{c,i})}{c_h(T_{h,i} - T_{h,o})} = \frac{5 \text{kg/s} \times 4181 \text{J/kg.K}(60 - 40)^\circ C}{4197 \text{J/kg.K}(95 - 60)^\circ C} = 2.85 \text{kg/s} \]

(b) From the rate equation and the log mean temperature relation,

\[ q = UA\Delta T_{lm,PF} \]

\[ \Delta T_{lm,PF} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} \]
And since $\Delta T_2=0$, $\Delta T_{lm}=0$ so that $UA=\infty$. Since $A=\pi DL$ is finite, $U$ must be extremely large. Hence, the heating cannot be accomplished with this arrangement.

(c) With the CF arrangements $m$ will be a minimum when $T_{ho}$ is a minimum. This requires that $T_{h,o}$ is a minimum. This requires that $T_{h,o}$ is a minimum. This requires that $T_{h,o}=T_{c,i}=40^\circ C$. Hence, from the overall energy balance,

$$m = \frac{5 \text{ kg/s} \times 4181 \text{ J/kg K} (60 - 40) \text{ K}}{4197 \text{ J/kg K} (95 - 40) \text{ K}} = 1.81 \text{ kg/s}$$

For this condition, $C_{min}=C_h$ which is cooled from $T_{h,i}$ to $T_{c,i}$, hence $\varepsilon=1$

Comments: For the counter flow arrangement, the heat exchanger must be infinitely long.
An automobile radiator may be viewed as a cross-flow heat exchanger with both fluids unmixed. Water, which has flow rate of 0.05kg/s, enters the radiator at 400K and is to leave at 330 K. The water is cooled by air which enters at 0.75kg/s and 300K. If the overall heat transfer coefficient is 200W/m².K, what is the required heat transfer surface area? Known: flow rate and inlet temperature for automobile radiator. Overall heat transfer coefficient. Find: Area required to achieve a prescribed outlet temperature.

Assumptions: (1) Negligible heat loss to surroundings and kinetic and potential energy changes, (2) Constant properties.
Analysis: The required heat transfer rate is

\[ q = (m \cdot c) (T_{h,i} - T_{h,o}) = 0.05 \text{ kg/s} (4209 \text{ J/kg.K}) \times 70 \text{ K} = 14,732 \text{ W} \]

Using the \( \varepsilon \)-NTU method,

\[
C_{\text{min}} = C_h = 210.45 \text{ W/K} \\
C_{\text{max}} = C_c = 755.25 \text{ W/K},
\]

\[
\text{hence, } C_{\text{min}} / C_{\text{max}} (T_{h,i} - T_{c,i}) = 210.45 \text{ W/K} (100 \text{ K}) = 21,045 \text{ W}
\]

\[
\varepsilon = q / q_{\text{max}} = 14,732 \text{ W} / 21,045 \text{ W} = 0.700
\]

From figure, NTU \( \approx 1.5 \), hence

\[
A = NTU(C_{\text{min}} / U) = 1.5 \times 210.45 \text{ W/K} (200 \text{ W/m}^2 \cdot \text{K}) = 1.58 \text{ m}^2
\]

Comments: (1) the air outlet temperature is

\[
T_{c,o} = T_{c,i} + q / C_c = 300 \text{ K} + (14,732 \text{ W} / 755.25 \text{ W/K}) = 319.5 \text{ K}
\]

(2) Using the LMTD approach, \( \Delta T_{\text{in}} = 51.2 \text{ K} \), \( R = 0.279 \) and \( P = 0.7 \). Hence from fig \( F \approx 0.95 \) and

\[
A = q / FU\Delta T_{\text{in}} = (14,732 \text{ W}) /[0.95(200 \text{ W/m}^2 \cdot \text{K})51.2 \text{ K}] = 1.51 \text{ m}^2.
\]
Water at 225 kg/h is to be heated from 35 to 95°C by means of a concentric tube heat exchanger. Oil at 225kg/h and 210°C, with a specific heat of 2095 J/kg.K, is to be used as the hot fluid. If the overall heat transfer coefficient based on the outer diameter of the inner tube is 550W/m².K, determine the length of the exchanger if the outer diameters is 100mm. 

**Known:** Concentric tube heat exchanger. Find: Length of the exchanger

**Assumptions:** (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

**Analysis:** From rate equation with \( A_o = \pi D_o L \), \( L = q/U_o D_o \Delta T \ell_m \)

The heat rate, \( q \), can be evaluated from an energy balance on the cold fluid:

\[
q = \dot{m}_c c_v (T_{c,0} - T_{c,i}) = \frac{225 \text{ kg/h}}{3600 \text{ s/h}} \times 4188 \text{ J/kg.K}(95 - 35) \text{ K} = 15,705 \text{ W}
\]
In order to evaluate $\Delta T_{\ell, m}$, we need to know whether the exchanger is operating in CF or PF. From an energy balance on the hot fluid, find

\[
T_{h,o} = T_{h,i} - \frac{q}{m_h c_h} = 210^\circ C - 15,705 W / \frac{225 kg}{h} \times \frac{3600 s}{h} \times 2095 \frac{J}{kg.K} = 90.1^\circ C
\]

Since $T_{h,o} < T_{c,o}$ it follows that HXer operation must be CF. From eq. for log mean temperature difference,

\[
\Delta T_{\ell,m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n(\Delta T_1 / \Delta T_2)} = \frac{(210 - 95) - (90.1 - 35)}{\ell n(115/55.1)} = 81.5^\circ C
\]

Substituting numerical values, the HXer length is

\[
L = 15,705 W / 550 W / m^2 \cdot K \pi(0.10 m) \times 81.4 K = 1.12 m
\]

\[
\varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}} = \frac{210 - 90.1}{210 - 35} = 0.69
\]
Consider a very long, concentric tube heat exchanger having hot and cold water inlet temperatures of 85 and 15°C. The flow rate of the hot water is twice that of the cold water. Assuming equivalent hot and cold water specifies heats; determine the hot water outlet temperature for the following modes of operation (a) Counter flow, (b) Parallel flow.

Known: A very long, concentric tube heat exchanger having hot and cold water inlet temperatures of 85 and 15°C, respectively: flow rate of the hot water is twice that of the cold water. Find: outlet temperatures for counter flow and parallel flow operations.

Assumptions: (1) equivalent hot and cold water specific heats, (2) Negligible Kinetic and potential energy changes, (3) No heat loss to surroundings.
Operating in the counter flow mode is:

\[ q = q_{\text{max}} = C_{\text{min}} (T_{h,i} - T_{c,i}) \]

Combining the above relation and rearranging, find

\[ T_{h,o} = -\frac{C_{\text{min}}}{C_h} (T_{h,i} - T_{c,i}) + T_{h,i} = -\frac{C_c}{C_h} (T_{h,i} - T_{c,i}) + T_{h,i} \]

Substituting numerical values:

\[ T_{h,o} = -\frac{1}{2} (85 - 15)°C + 85°C = 50°C \]

For parallel flow operation, the hot and cold outlet temperatures will be equal; that is \( T_{c,o} = T_{h,o} \). Hence

\[ C_c (T_{c,o} - T_{c,i}) = C_h (T_{h,i} - T_{h,o}) \]
Setting $T_{c,o}=T_{h,o}$ and rearranging

$$T_{h,o} = \left[ T_{h,i} + \frac{C_c}{C_h} T_{c,i} \right] \left/ \left[ 1 + \frac{C_c}{C_h} \right] \right.$$

$$T_{h,o} = \left[ 85 + \frac{1}{2} \times 15 \right] ^\circ C \left/ \left[ 1 + \frac{1}{2} \right] \right. = 61.7 ^\circ C$$

Comments: Note that while $\varepsilon =1$ for CF operation, for PF operation find $\varepsilon = \frac{q}{q_{\text{max}}}=0.67.$
Maximum heat transfer

The heat transfer in a heat exchanger will reach its maximum value when

(1) the cold fluid is heated to the inlet temperature of the hot fluid or
(2) The hot fluid is cooled to the inlet temperature of the cold fluid.

These two limiting conditions will not be reached simultaneously unless the heat capacity rates of the hot and cold fluids are identical (i.e., $C_c = C_h$). When $C_c \neq C_h$, which is usually the case, the fluid with the smaller heat capacity rate will experience a larger temperature change, and thus it will be the first to experience the maximum temperature, at which point the heat transfer will come to a halt.

Maximum possible heat transfer

$$\dot{Q}_{\text{max}} = C_{\text{min}}(T_{h,\text{in}} - T_{c,\text{in}})$$

where $C_{\text{min}}$ is the smaller of $C_h = \dot{m}_h C_{ph}$ and $C_c = \dot{m}_c C_{pc}$
**EXAMPLE 23–7 Upper Limit for Heat Transfer in a Heat Exchanger**

Cold water enters a counter-flow heat exchanger at 10°C at a rate of 8 kg/s, where it is heated by a hot-water stream that enters the heat exchanger at 70°C at a rate of 2 kg/s. Assuming the specific heat of water to remain constant at \( C_p = 4.18 \text{ kJ/kg \cdot °C} \), determine the maximum heat transfer rate and the outlet temperatures of the cold- and the hot-water streams for this limiting case.

**SOLUTION** Cold- and hot-water streams enter a heat exchanger at specified temperatures and flow rates. The maximum rate of heat transfer in the heat exchanger is to be determined.

**Assumptions**  
1. Steady operating conditions exist.  
2. The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid.  
3. Changes in the kinetic and potential energies of fluid streams are negligible.  
4. Heat transfer coefficients and fouling factors are constant and uniform.  
5. The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

**Properties** The specific heat of water is given to be \( C_p = 4.18 \text{ kJ/kg \cdot °C} \).

**Analysis** A schematic of the heat exchanger is given in Fig. 23–24. The heat capacity rates of the hot and cold fluids are determined from

\[
C_h = \dot{m}_h C_{ph} = (2 \text{ kg/s})(4.18 \text{ kJ/kg \cdot °C}) = 8.36 \text{ kW/°C}
\]

and

\[
C_c = \dot{m}_c C_{pc} = (8 \text{ kg/s})(4.18 \text{ kJ/kg \cdot °C}) = 33.4 \text{ kW/°C}
\]
Therefore \( C_{\text{min}} = C_h = 8.36 \text{ kW/°C} \)

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate is determined from Eq. 23–32 to be

\[
\dot{Q}_{\text{max}} = C_{\text{min}}(T_{h,\text{in}} - T_{c,\text{in}}) \\
= (8.36 \text{ kW/°C})(70 - 10)\text{°C} \\
= 502 \text{ kW}
\]

That is, the maximum possible heat transfer rate in this heat exchanger is 502 kW. This value would be approached in a counter-flow heat exchanger with a very large heat transfer surface area.

The maximum temperature difference in this heat exchanger is \( \Delta T_{\text{max}} = T_{h,\text{in}} - T_{c,\text{in}} = (70 - 10)\text{°C} = 60\text{°C} \). Therefore, the hot water cannot be cooled by more than 60°C (to 10°C) in this heat exchanger, and the cold water cannot be heated by more than 60°C (to 70°C), no matter what we do. The outlet temperatures of the cold and the hot streams in this limiting case are determined to be

\[
\dot{Q} = C_c(T_{c,\text{out}} - T_{c,\text{in}}) \quad \rightarrow \quad T_{c,\text{out}} = T_{c,\text{in}} + \frac{\dot{Q}}{C_c} = 10\text{°C} + \frac{502 \text{ kW}}{33.4 \text{ kW/°C}} = 25\text{°C}
\]

\[
\dot{Q} = C_h(T_{h,\text{in}} - T_{h,\text{out}}) \quad \rightarrow \quad T_{h,\text{out}} = T_{h,\text{in}} - \frac{\dot{Q}}{C_h} = 70\text{°C} - \frac{502 \text{ kW}}{8.38 \text{ kW/°C}} = 10\text{°C}
\]
The determination of $Q_{\text{max}}$ requires the availability of the inlet temperature of the hot and cold fluids and their mass flow rates, which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate can be determined from:

$$\dot{Q} = \varepsilon \dot{Q}_{\text{max}} = \varepsilon C_{\text{min}}(T_{\text{h, in}} - T_{\text{c, in}})$$

The below equation developed before in previous lecture for a parallel-flow heat exchanger:

$$\ln \frac{T_{\text{h, out}} - T_{\text{c, out}}}{T_{\text{h, in}} - T_{\text{c, in}}} = -UA_s \left( \frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right)$$

can be rearranged as:

$$\ln \frac{T_{\text{h, out}} - T_{\text{c, out}}}{T_{\text{h, in}} - T_{\text{c, in}}} = -\frac{UA_s}{C_c} \left( 1 + \frac{C_c}{C_h} \right)$$

Also, solving the following Eq. for $T_{\text{h, out}}$

$$\dot{Q} = C_c(T_{\text{c, out}} - T_{\text{c, in}}) = C_h(T_{\text{h, in}} - T_{\text{h, out}})$$

Gives:

$$T_{\text{h, out}} = T_{\text{h, in}} - \frac{C_c}{C_h} (T_{\text{c, out}} - T_{\text{c, in}})$$
Then:

\[
\frac{T_{h,\text{in}} - T_{c,\text{in}} + T_{c,\text{in}} - T_{c,\text{out}} - \frac{C_c}{C_h}(T_{c,\text{out}} - T_{c,\text{in}})}{\ln \frac{T_{h,\text{in}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}}} = -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right)
\]

which simplifies to:

\[
\ln \left[1 - \left(1 + \frac{C_c}{C_h}\right) \frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}}\right] = -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right)
\]

We now manipulate the definition of effectiveness to obtain:

\[
\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} = \frac{C_c(T_{c,\text{out}} - T_{c,\text{in}})}{C_{\text{min}}(T_{h,\text{in}} - T_{c,\text{in}})} \quad \rightarrow \quad \frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \varepsilon \frac{C_{\text{min}}}{C_c}
\]

Substituting this result into Eq. 23–36 and solving for \( \varepsilon \) gives the following relation for the effectiveness of a parallel-flow heat exchanger:

\[
\varepsilon_{\text{parallel flow}} = 1 - \exp \left[-\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right)\right]
\]

\[
\varepsilon_{\text{parallel flow}} = \frac{C_c}{\left(1 + \frac{C_c}{C_h}\right) \frac{C_{\text{min}}}{C_c}}
\]

Taking either \( C_c \) or \( C_h \) to be \( C_{\text{min}} \) (both approaches give the same result), the relation above can be expressed more conveniently as:

\[
\varepsilon_{\text{parallel flow}} = 1 - \exp \left[-\frac{UA_s}{C_{\text{min}}} \left(1 + \frac{C_{\text{min}}}{C_{\text{max}}}\right)\right]
\]

\[
\varepsilon_{\text{parallel flow}} = \frac{1}{1 + \frac{C_{\text{min}}}{C_{\text{max}}}}
\]
Effectiveness relations of the heat exchangers typically involve the dimensionless group $U_{A_s}/C_{\text{min}}$. This quantity is called the number of transfer units NTU and is expressed as:

$$\text{NTU} = \frac{U A_s}{C_{\text{min}}} = \frac{U A_s}{(mC_p)_{\text{min}}}$$

where $U$ is the overall heat transfer coefficient and $A_s$ is the heat transfer surface area of the heat exchanger.

NTU is a measure of the heat transfer surface area $A_s$. Thus, the larger the NTU, the larger the heat exchanger.

In heat exchanger analysis, it is also convenient to define another dimensionless quantity called the capacity ratio $c$ as:

$$c = \frac{C_{\text{min}}}{C_{\text{max}}}$$
Effectiveness relations have been developed for a large number of heat exchangers, and the results are given in the Table below. The effectivenesses of some common types of heat exchangers are also plotted in the becoming figures.

<table>
<thead>
<tr>
<th>Heat exchanger type</th>
<th>Effectiveness relation</th>
</tr>
</thead>
</table>
| **1 Double pipe:**  | **Effectiveness relations for heat exchangers: NTU = UA_s/C_{min} and**  
| Parallel-flow       | $\varepsilon = \frac{1 - \exp [-NTU(1 + c)]}{1 + c}$  
| Counter-flow        | $\varepsilon = \frac{1 - \exp [-NTU(1 - c)]}{1 - c \exp [-NTU(1 - c)]}$  
| **2 Shell-and-tube:** | **Effectiveness relations for heat exchangers: NTU = UA_s/C_{min} and**  
| One-shell pass      | $\varepsilon = 2 \left(1 + c + \sqrt{1 + c^2} \frac{1 + \exp [-NTU\sqrt{1 + c^2}]}{1 - \exp [-NTU\sqrt{1 + c^2}]}\right)^{-1}$  
| 2, 4, ... tube passes |                      |
| **3 Cross-flow**    | **Effectiveness relations for heat exchangers: NTU = UA_s/C_{min} and**  
| (single-pass)       | $\varepsilon = 1 - \exp \left\{ \frac{NTU^{0.22}}{c} \left[\exp (-c \ NTU^{0.78}) - 1\right]\right\}$  
| Both fluids unmixed | $\varepsilon = \frac{1}{c} (1 - \exp \{1-c[1 - \exp (-NTU)]\})$  
| $C_{max}$ mixed, $C_{min}$ unmixed | $\varepsilon = 1 - \exp \left\{-\frac{1}{c} [1 - \exp (-c \ NTU)]\right\}$  
| $C_{min}$ mixed, $C_{max}$ unmixed | $\varepsilon = 1 - \exp(-NTU)$  
| **4 All heat exchangers with c = 0** | $\varepsilon = 1 - \exp(-NTU)$  

**TABLE 23-4**
Effectiveness for heat exchangers (from Kays and London).
FIGURE 23–26
Effectiveness for heat exchangers (from Kays and London).

(c) One-shell pass and 2, 4, 6, ... tube passes

(d) Two-shell passes and 4, 8, 12, ... tube passes
(e) Cross-flow with both fluids unmixed

(f) Cross-flow with one fluid mixed and the other unmixed

FIGURE 23–26
Effectiveness for heat exchangers (from Kays and London).
Some observations from the effectiveness relations and charts already given:

1. The value of the effectiveness ranges from 0 to 1. It increases rapidly with NTU for small values (up to about NTU 1.5) but rather slowly for larger values. Therefore, the use of a heat exchanger with a large NTU (usually larger than 3) and thus a large size cannot be justified economically, since a large increase in NTU in this case corresponds to a small increase in effectiveness. Thus, a heat exchanger with a very high effectiveness may be highly desirable from a heat transfer point of view but rather undesirable from an economical point of view.

2. For a given NTU and capacity ratio $c = C_{\text{min}} / C_{\text{max}}$, the counter-flow heat exchanger has the highest effectiveness, followed closely by the cross-flow heat exchangers with both fluids unmixed. As you might expect, the lowest effectiveness values are encountered in parallel-flow heat exchangers (Fig. 23–27).
3. The effectiveness of a heat exchanger is independent of the capacity ratio c for NTU values of less than about 0.3.

4. The value of the capacity ratio c ranges between 0 and 1. For a given NTU, the effectiveness becomes a maximum for c = 0 and a minimum for c = 1.

The case $c = \frac{C_{\text{min}}}{C_{\text{max}}} \rightarrow 0$ corresponds to $C_{\text{max}} \rightarrow \infty$, which is realized during a phase-change process in a condenser or boiler. All effectiveness relations in this case reduce to:

$$
\varepsilon = \varepsilon_{\text{max}} = 1 - \exp(-\text{NTU})
$$
it can also be determined from the effectiveness–NTU method by first evaluating the effectiveness $\varepsilon$ from its definition

$$\varepsilon = \frac{\dot{Q}}{Q_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

and then the NTU from the appropriate NTU relation mentioned in the following Table.

<table>
<thead>
<tr>
<th>Heat exchanger type</th>
<th>NTU relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double-pipe:</td>
<td>NTU = $-\frac{\ln [1 - \varepsilon(1 + c)]}{1 + c}$</td>
</tr>
<tr>
<td>Parallel-flow</td>
<td>NTU = $\frac{1}{c - 1} \ln \left( \frac{\varepsilon - 1}{\varepsilon c - 1} \right)$</td>
</tr>
<tr>
<td>Counter-flow</td>
<td>NTU = $-\frac{1}{\sqrt{1 + c^2}} \ln \left( \frac{2/e - 1 - c - \sqrt{1 + c^2}}{2/e - 1 - c + \sqrt{1 + c^2}} \right)$</td>
</tr>
<tr>
<td>Shell and tube:</td>
<td>NTU = $-\ln \left[ 1 + \frac{\ln (1 - \varepsilon c)}{c} \right]$</td>
</tr>
<tr>
<td>One-shell pass</td>
<td>NTU = $-\ln \left[ c \ln (1 - \varepsilon) + 1 \right]$</td>
</tr>
<tr>
<td>2, 4, ... tube passes</td>
<td>NTU = $-\ln(1 - \varepsilon)$</td>
</tr>
</tbody>
</table>

**TABLE 23–5**

NTU relations for heat exchangers: $\text{NTU} = 
\frac{\dot{Q}}{Q_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$

$\varepsilon = \frac{\dot{Q}}{Q_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$

$\text{NTU} = \frac{1}{c - 1} \ln \left( \frac{\varepsilon - 1}{\varepsilon c - 1} \right)$
EXAMPLE Using the Effectiveness–NTU Method

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m² · °C, determine the length of the heat exchanger required to achieve the desired heating.

Repeat Example 23–4, which was solved with the LMTD method, using the effectiveness–NTU method.

SOLUTION The schematic of the heat exchanger is redrawn in Fig. 23–29, and the same assumptions are utilized.

Analysis In the effectiveness–NTU method, we first determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

\[ C_h = m_h C_{ph} = (2 \text{ kg/s})(4.31 \text{ kJ/kg} \cdot \text{ °C}) = 8.62 \text{ kW/°C} \]
\[ C_c = m_c C_{pc} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot \text{ °C}) = 5.02 \text{ kW/°C} \]

Therefore,

\[ C_{\text{min}} = C_c = 5.02 \text{ kW/°C} \]

and

\[ c = C_{\text{min}}/C_{\text{max}} = 5.02/8.62 = 0.583 \]

Then the maximum heat transfer rate is determined from Eq. 23–32 to be

\[ \dot{Q}_{\text{max}} = C_{\text{min}}(T_{h,\text{in}} - T_{c,\text{in}}) \]
\[ = (5.02 \text{ kW/°C})(160 - 20)\text{ °C} \]
\[ = 702.8 \text{ kW} \]
That is, the maximum possible heat transfer rate in this heat exchanger is 702.8 kW. The actual rate of heat transfer in the heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{water} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot \text{°C})(80 - 20)\text{°C} = 301.0 \text{ kW}$$

Thus, the effectiveness of the heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{301.0 \text{ kW}}{702.8 \text{ kW}} = 0.428$$

Knowing the effectiveness, the NTU of this counter-flow heat exchanger can be determined from Fig. 23–26b or the appropriate relation from Table 23–5. We choose the latter approach for greater accuracy:

$$\text{NTU} = \frac{1}{c - 1} \ln \left( \frac{\varepsilon - 1}{\varepsilon c - 1} \right) = \frac{1}{0.583 - 1} \ln \left( \frac{0.428 - 1}{0.428 \times 0.583 - 1} \right) = 0.651$$

Then the heat transfer surface area becomes

$$\text{NTU} = \frac{UA_s}{C_{min}} \quad \rightarrow \quad A_s = \frac{\text{NTU} C_{min}}{U} = \frac{(0.651)(5020 \text{ W/°C})}{640 \text{ W/m}^2 \cdot \text{°C}} = 5.11 \text{ m}^2$$

To provide this much heat transfer surface area, the length of the tube must be

$$A_s = \pi DL \quad \rightarrow \quad L = \frac{A_s}{\pi D} = \frac{5.11 \text{ m}^2}{\pi(0.015 \text{ m})} = 108 \text{ m}$$
 EXAMPLE: Cooling Hot Oil by Water in a Multipass Heat Exchanger

Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 W/m² · °C. Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20°C and 150°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.

**SOLUTION** Hot oil is to be cooled by water in a heat exchanger. The mass flow rates and the inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. 3 The thickness of the tube is negligible since it is thin-walled. 4 Changes in the kinetic and potential energies of fluid streams are negligible. 5 The overall heat transfer coefficient is constant and uniform.

**Analysis** The schematic of the heat exchanger is given in Fig. 23–30. The outlet temperatures are not specified, and they cannot be determined from an energy balance. The use of the LMTD method in this case will involve tedious iterations, and thus the ε–NTU method is indicated. The first step in the ε–NTU method is to determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

\[ C_h = \dot{m}_h C_{ph} = (0.3 \text{ kg/s})(2.13 \text{ kJ/kg} \cdot \text{°C}) = 0.639 \text{ kW/°C} \]
\[ C_c = \dot{m}_c C_{pc} = (0.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot \text{°C}) = 0.836 \text{ kW/°C} \]
Therefore, \( C_{\text{min}} = C_h = 0.639 \text{ kW/°C} \)

and \( c = \frac{C_{\text{min}}}{C_{\text{max}}} = \frac{0.639}{0.836} = 0.764 \)

Then the maximum heat transfer rate is determined from Eq. 23–32 to be

\[
\dot{Q}_{\text{max}} = C_{\text{min}}(T_{h,\text{in}} - T_{c,\text{in}}) \\
= (0.639 \text{ kW/°C})(150 - 20)°C = 83.1 \text{ kW}
\]

That is, the maximum possible heat transfer rate in this heat exchanger is 83.1 kW. The heat transfer surface area is

\[
A_s = n(\pi DL) = 8\pi(0.014 \text{ m})(5 \text{ m}) = 1.76 \text{ m}^2
\]

Then the NTU of this heat exchanger becomes

\[
\text{NTU} = \frac{UA_s}{C_{\text{min}}} = \frac{(310 \text{ W/m}^2 \cdot °C)(1.76 \text{ m}^2)}{639 \text{ W/°C}} = 0.853
\]

The effectiveness of this heat exchanger corresponding to \( c = 0.764 \) and NTU = 0.853 is determined from Fig. 23–26c to be

\[\varepsilon = 0.47\]

We could also determine the effectiveness from the third relation in Table 23–4 more accurately but with more labor. Then the actual rate of heat transfer becomes

\[
\dot{Q} = \varepsilon \dot{Q}_{\text{max}} = (0.47)(83.1 \text{ kW}) = 39.1 \text{ kW}
\]
Finally, the outlet temperatures of the cold and the hot fluid streams are determined to be

\[
\dot{Q} = C_c(T_{c,\text{out}} - T_{c,\text{in}}) \quad \rightarrow \quad T_{c,\text{out}} = T_{c,\text{in}} + \frac{\dot{Q}}{C_c}
\]

\[
= 20^\circ C + \frac{39.1 \text{ kW}}{0.836 \text{ kW/}^\circ C} = 66.8^\circ C
\]

\[
\dot{Q} = C_h(T_{h,\text{in}} - T_{h,\text{out}}) \quad \rightarrow \quad T_{h,\text{out}} = T_{h,\text{in}} - \frac{\dot{Q}}{C_h}
\]

\[
= 150^\circ C - \frac{39.1 \text{ kW}}{0.639 \text{ kW/}^\circ C} = 88.8^\circ C
\]

Therefore, the temperature of the cooling water will rise from 20°C to 66.8°C as it cools the hot oil from 150°C to 88.8°C in this heat exchanger.
Number of Transfer Units (NTU)

Recall that the energy flow in any HX is described by three equations:

\[
Q = U \cdot A \cdot \Delta \theta_{\text{eff}} \quad \text{HX equation}
\]

\[
Q = -M \cdot C_p \cdot \Delta T \quad \text{1st Law Equation}
\]

\[
Q = m \cdot c_p \cdot \Delta t \quad \text{1st Law Equation}
\]

We may generalize the latter two expressions, using \(\varepsilon\)-NTU terminology as follows:

\[
Q = (M \cdot C_p)_{\text{min}} \cdot \Delta T_{\text{max}}
\]

\[
Q = (M \cdot C_p)_{\text{max}} \cdot \Delta T_{\text{min}}
\]

If we eliminate \(Q\) between the HX equation and one of the 1st Law equations:

\[
U \cdot A \cdot \Delta \theta_{\text{eff}} = (M \cdot C_p)_{\text{min}} \cdot \Delta T_{\text{max}}
\]

This expression may be made non-dimensional by taking the temperatures to one side and the other terms to the other side:

\[
NTU \equiv \frac{U \cdot A}{(\dot{M} \cdot C_p)_{\text{min}}} = \frac{\Delta T_{\text{max}}}{\Delta \theta_{\text{eff}}}
\]
Problem

- An 25 m$^2$ counterflow heat exchanger with $U = 200$ W/m$^2$·°C is to be used to cool 1 kg/s of oil ($c_p = 2000$ J/kg·°C) at 100°C using 3 kg/s of water ($c_p = 4184$ J/kg·°C) at 20°C. What is the oil outlet temperature.

- **Given:** $T_{h,in} = 100$°C, $T_{c,in} = 20$°C, $U = 200$ W/m$^2$·°C, $A = 25$ m$^2$, $c_{p,c} = 4184$ J/kg·°C, $c_{p,h} = 2000$ J/kg·°C, $m_h = 1$ kg/s, and $m_c = 3$ kg/s. **Find:** $T_{h,out}$
Solution

\[ C_h = \dot{m}_h c_{ph} = \frac{1 \text{ kg}}{s} \frac{2000 \text{ J}}{\text{kg} \cdot ^\circ \text{C}} = \frac{2000 \text{ J}}{s \cdot ^\circ \text{C}} \]

\[ C_c = \dot{m}_c c_{pc} = \frac{3 \text{ kg}}{s} \frac{4184 \text{ J}}{\text{kg} \cdot ^\circ \text{C}} = \frac{12552 \text{ J}}{s \cdot ^\circ \text{C}} \]

\[ C_{\text{min}} = \frac{2000 \text{ J}}{s \cdot ^\circ \text{C}} \]

\[ NTU = \frac{UA}{C_{\text{min}}} = \frac{200 \text{ W}}{m^2 \cdot ^\circ \text{C}} \frac{(25 \text{ m}^2)}{2000 \text{ J} \frac{W \cdot s}{s \cdot ^\circ \text{C}}} = 2.5 \]

\[ c = \frac{C_{\text{min}}}{C_{\text{max}}} = \frac{\frac{2000 \text{ J}}{s \cdot ^\circ \text{C}}}{\frac{12552 \text{ J}}{s \cdot ^\circ \text{C}}} = 0.1593 \]
Chart $\varepsilon$

From chart for counter flow:

$NTU = 2.5$

$\frac{C_{\text{min}}}{C_{\text{max}}} = 0.16$

$\varepsilon = 0.89$
Effectiveness Equation

- For counterflow heat exchangers \( (c = \frac{C_{\text{min}}}{C_{\text{max}}}) \)
  \[
  \varepsilon = \frac{1 - e^{-NTU(1-c)}}{1 - ce^{-NTU(1-c)}}
  \]
  \[
  \varepsilon = \frac{1 - e^{-2.5(1-0.1593)}}{1 - 0.1593e^{-2.5(1-0.1593)}} = 0.895
  \]

\[
\dot{Q} = \varepsilon \dot{Q}_{\text{max}} = \varepsilon C_{\text{min}} (T_{h,\text{in}} - T_{c,\text{in}}) = 0.895 \frac{2000 J}{s \cdot ^\circ C} (100^\circ C - 20^\circ C)
\]

\[
\dot{Q} = 1.43 \times 10^5 \frac{J}{s} = 143 \text{ kW}
\]
Outlet Temperatures

- Use basic energy balance equations to find outlet temperatures from $Q$

\[
T_{c,\text{out}} = T_{c,\text{in}} + \frac{\dot{Q}}{m_c c_{p_c}} = 20^\circ C + \frac{1.43 \times 10^5 \frac{J}{s}}{3 \text{ kg} \cdot \frac{4184 \text{ J}}{\text{kg} \cdot \circ C}} = 31.4^\circ C
\]

\[
T_{h,\text{out}} = T_{h,\text{in}} - \frac{\dot{Q}}{m_c c_{p_c}} = 100^\circ C - \frac{1.43 \times 10^5 \frac{J}{s}}{3 \text{ kg} \cdot \frac{4184 \text{ J}}{\text{kg} \cdot \circ C}} = 28.4^\circ C
\]
The final non-dimensional ratio needed here is the capacity ratio, defined as follows:

\[ C_R \equiv \frac{(M \cdot C_p)_{\text{min}}}{(M \cdot C_p)_{\text{max}}} = \frac{\Delta T_{\text{min}}}{\Delta T_{\text{max}}} \]

In the LMTD-F method a capacity ratio was defined:

\[ R = \frac{T_1 - T_2}{t_2 - t_1} \]
SELECTION OF HEAT EXCHANGERS

Heat exchangers are complicated devices, and the results obtained with the simplified approaches presented above should be used with care. The proper selection depends on several factors:

1. Heat Transfer Rate:
   A heat exchanger should be capable of transferring heat at the specified rate in order to achieve the desired temperature change of the fluid at the specified mass flow rate.

2. Cost:
   Budgetary limitations usually play an important role in the selection of heat exchangers, except for some specialized cases where “money is no object.”

3. Pumping Power:
   In a heat exchanger, both fluids are usually forced to flow by pumps or fans that consume electrical power. The annual cost of electricity associated with the operation of the pumps and fans can be determined from:

   \[\text{Operating cost} = (\text{Pumping power, kW}) \times (\text{Hours of operation, h}) \times (\text{Price of electricity, $/kWh})\]
4. Size and Weight:
Normally, the smaller and the lighter the heat exchanger, the better it is.

5. Type:
The type of heat exchanger to be selected depends primarily on the type of fluids involved, the size and weight limitations, and the presence of any phase change processes.

6. Materials:
The materials used in the construction of the heat exchanger may be an important consideration in the selection of heat exchangers. For example, the thermal and structural stress effects need not be considered at pressures below 15 atm or temperatures below 150°C.
EXAMPLE 23–10 Installing a Heat Exchanger to Save Energy and Money

In a dairy plant, milk is pasteurized by hot water supplied by a natural gas furnace. The hot water is then discharged to an open floor drain at 80°C at a rate of 15 kg/min. The plant operates 24 h a day and 365 days a year. The furnace has an efficiency of 80 percent, and the cost of the natural gas is $0.40 per therm (1 therm = 105,500 kJ). The average temperature of the cold water entering the furnace throughout the year is 15°C. The drained hot water cannot be returned to the furnace and recirculated, because it is contaminated during the process.

In order to save energy, installation of a water-to-water heat exchanger to preheat the incoming cold water by the drained hot water is proposed. Assuming that the heat exchanger will recover 75 percent of the available heat in the hot water, determine the heat transfer rating of the heat exchanger that needs to be purchased and suggest a suitable type. Also, determine the amount of money this heat exchanger will save the company per year from natural gas savings.

SOLUTION A water-to-water heat exchanger is to be installed to transfer energy from drained hot water to the incoming cold water to preheat it. The rate of heat transfer in the heat exchanger and the amount of energy and money saved per year are to be determined.

Assumptions 1 Steady operating conditions exist. 2 The effectiveness of the heat exchanger remains constant.

Properties We use the specific heat of water at room temperature, \( c_p = 4.18 \) kJ/kg \( \cdot \) °C, and treat it as a constant.

Analysis A schematic of the prospective heat exchanger is given in Fig. 23–31. The heat recovery from the hot water will be a maximum when it leaves the heat exchanger at the inlet temperature of the cold water. Therefore,
\[ \dot{Q}_{\text{max}} = \dot{m}_h C_p (T_{h, \text{in}} - T_{c, \text{in}}) \]
\[ = \left( \frac{15}{60} \text{ kg/s} \right) (4.18 \text{ kJ/kg} \cdot ^\circ \text{C}) (80 - 15)^\circ \text{C} \]
\[ = 67.9 \text{ kJ/s} \]

That is, the existing hot-water stream has the potential to supply heat at a rate of 67.9 kJ/s to the incoming cold water. This value would be approached in a counter-flow heat exchanger with a very large heat transfer surface area. A heat exchanger of reasonable size and cost can capture 75 percent of this heat transfer potential. Thus, the heat transfer rating of the prospective heat exchanger must be
\[ \dot{Q} = \varepsilon \dot{Q}_{\text{max}} = (0.75)(67.9 \text{ kJ/s}) = 50.9 \text{ kJ/s} \]

That is, the heat exchanger should be able to deliver heat at a rate of 50.9 kJ/s from the hot to the cold water. An ordinary plate or shell-and-tube heat exchanger should be adequate for this purpose, since both sides of the heat exchanger involve the same fluid at comparable flow rates and thus comparable heat transfer coefficients. (Note that if we were heating air with hot water, we would have to specify a heat exchanger that has a large surface area on the air side.)

The heat exchanger will operate 24 h a day and 365 days a year. Therefore, the annual operating hours are

Operating hours = (24 h/day)(365 days/year) = 8760 h/year
Noting that this heat exchanger saves 50.9 kJ of energy per second, the energy saved during an entire year will be

\[
\text{Energy saved} = (\text{Heat transfer rate})(\text{Operation time}) \\
= (50.9 \text{ kJ/s})(8760 \text{ h/year})(3600 \text{ s/h}) \\
= 1.605 \times 10^9 \text{ kJ/year}
\]

The furnace is said to be 80 percent efficient. That is, for each 80 units of heat supplied by the furnace, natural gas with an energy content of 100 units must be supplied to the furnace. Therefore, the energy savings determined above result in fuel savings in the amount of

\[
\text{Fuel saved} = \frac{\text{Energy saved}}{\text{Furnace efficiency}} = \frac{1.605 \times 10^9 \text{ kJ/year}}{0.80} \left( \frac{1 \text{ therm}}{105,500 \text{ kJ}} \right) \\
= 19,020 \text{ therms/year}
\]

Noting that the price of natural gas is $0.40 per therm, the amount of money saved becomes

\[
\text{Money saved} = (\text{Fuel saved}) \times (\text{Price of fuel}) \\
= (19,020 \text{ therms/year})(0.40/\text{therm}) \\
= $7607/\text{year}
\]

Therefore, the installation of the proposed heat exchanger will save the company $7607 a year, and the installation cost of the heat exchanger will probably be paid from the fuel savings in a short time.