Chapter 7

Minor Losses
Minor Losses

- Most pipe systems consist of considerably more than straight pipes. These additional components (valves, bends, tees, and the like) add to the overall head loss of the system.
- Such losses are termed **MINOR LOSSES**.

The flow pattern through a valve
90° Bend

90° Elbow

45° Elbow

Tees-line flow

Tees-branch flow
VALVES

Globe valve

Angle valve

Gate valve

Ball valve
Minor Losses

- The theoretical analysis to predict the details of flow pattern (through these additional components) is not, as yet, possible.
- The head loss information for essentially all components is given in dimensionless form and based on experimental data.
- The most common method used to determine these head losses or pressure drops is to specify the loss coefficient, \( K_L \)
Minor Losses

\[ K_L = \frac{h_{L_{\text{min or}}}}{V^2 / 2g} = \frac{\Delta p}{\frac{1}{2} \rho V^2} \Rightarrow \Delta p = K_L \frac{1}{2} \rho V^2 \]

Minor losses are sometimes given in terms of an equivalent length \( \ell_{eq} \) which is the length of the same pipe that would give the same pressure loss.

\[ h_{L_{\text{min or}}} = K_L \frac{V^2}{2g} = f \frac{\ell_{eq}}{D} \frac{V^2}{2g} \]

\[ \ell_{eq} = K_L \frac{D}{f} \]

The actual value of \( K_L \) is strongly dependent on the geometry of the component considered. It may also dependent on the fluid properties. That is

\[ K_L = \phi(\text{geometry, Re}) \]
Minor Losses

- Thus, in most cases of practical interest the loss coefficients for components are a function of geometry only,

\[ K_L = \phi(\text{geometry}) \]
## Minor Losses

<table>
<thead>
<tr>
<th>Component</th>
<th>( K_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Elbows</strong></td>
<td></td>
</tr>
<tr>
<td>Regular 90°, flanged</td>
<td>0.3</td>
</tr>
<tr>
<td>Regular 90°, threaded</td>
<td>1.5</td>
</tr>
<tr>
<td>Long radius 90°, flanged</td>
<td>0.2</td>
</tr>
<tr>
<td>Long radius 90°, threaded</td>
<td>0.7</td>
</tr>
<tr>
<td>Long radius 45°, flanged</td>
<td>0.2</td>
</tr>
<tr>
<td>Regular 45°, threaded</td>
<td>0.4</td>
</tr>
<tr>
<td><strong>180° return bends</strong></td>
<td></td>
</tr>
<tr>
<td>180° return bend, threaded</td>
<td>0.2</td>
</tr>
<tr>
<td>180° return bend, flanged</td>
<td>1.5</td>
</tr>
<tr>
<td><strong>Tees</strong></td>
<td></td>
</tr>
<tr>
<td>Line flow, flanged</td>
<td>0.2</td>
</tr>
<tr>
<td>Line flow, threaded</td>
<td>0.9</td>
</tr>
<tr>
<td>Branch flow, flanged</td>
<td>1.0</td>
</tr>
<tr>
<td>Branch flow, threaded</td>
<td>2.0</td>
</tr>
<tr>
<td><strong>Union, threaded</strong></td>
<td>0.8</td>
</tr>
<tr>
<td><strong>Valves</strong></td>
<td></td>
</tr>
<tr>
<td>Globe, fully open</td>
<td>10</td>
</tr>
<tr>
<td>Angle, fully open</td>
<td>2</td>
</tr>
<tr>
<td>Gate, fully open</td>
<td>0.15</td>
</tr>
<tr>
<td>Gate, ¼ closed</td>
<td>0.26</td>
</tr>
<tr>
<td>Gate, ½ closed</td>
<td>2.1</td>
</tr>
<tr>
<td>Gate, ¾ closed</td>
<td>17</td>
</tr>
<tr>
<td>Ball valve, fully open</td>
<td>0.05</td>
</tr>
<tr>
<td>Ball valve, 1/3 closed</td>
<td>5.5</td>
</tr>
<tr>
<td>Ball valve, 2/3 closed</td>
<td>210</td>
</tr>
</tbody>
</table>
Minor Losses

The mechanical energy equation can be written:

\[
\frac{P_1}{\rho g} - \frac{P_2}{\rho g} + \left( \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right) + (z_1 - z_2) + \frac{W_{\text{shaft}}}{\dot{m}g} = \\
\left( f \frac{L V^2}{D 2g} \right) + \sum K_L \frac{V^2}{2g}
\]

(6.22)
Minor Losses

➤ Method 2:
Using the concept of equivalent length, which is the equivalent length of pipe which would have the same friction effect as the fitting.

\[
\left( \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \right) + \left( \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right) + (z_1 - z_2) + \frac{W_{shaft}}{\dot{m}g} = \left( f \left( \frac{L}{D} \right)_{equiv.} \frac{V^2}{2g} \right)
\]

(6.23)

Values of equivalent length (L/D) can be found in the literature.
Pipe flow scenarios:

- The real world pipe flow design scenarios are divided into 3 types, I, II and III.
- **Type I:** We know the fluid, pipe size and desired flow rate. We need to determine the pressure drop or head loss. In effect we want to know how large a pump needs to be installed.
- **Type II:** We know the fluid, pipe size and head loss. We want to determine the flow rate.
- **Type III:** We know the fluid, flow rate and head loss. We want to determine pipe size.
- Type II and III scenarios are more complicated to solve since they involve a non-linear equation. Need to adjust $v$ and there is complicated dependence between $f$ and $v$. 
Example: Determine Pressure Drop

- Water at 60°F flows from the basement to the second floor through 0.75-in. diameter copper pipe (a drawn tubing) at a rate of $Q = 12.0 \text{ gal/min} = 0.0267 \text{ ft}^3/\text{s}$ and exits through a faucet of diameter 0.50 in. as shown in Figure

Determine the pressure at point (1) if: (a) all losses are neglected, (b) the only losses included are major losses, or (c) all losses are included.
Type I example: continued

Estimate fluid velocity to get flow rate.

\[ v_1 = \frac{Q}{A_1} = \frac{7.57 \times 10^{-4}}{\pi(9.5 \times 10^{-3})^2} = 2.67 \text{ m/s} \]

The Reynolds number is

\[ \text{Re} = \frac{\rho D v}{\mu} = \frac{10^3 \times 19.0 \times 10^{-3} \times 2.67}{1.12 \times 10^{-3}} = 45300 \]

The flow is turbulent flow. The equation to be applied is

\[ \frac{p_1}{\gamma} + \alpha_1 \frac{v_1^2}{2g} + z_1 - h_L = \frac{p_2}{\gamma} + \alpha_2 \frac{v_2^2}{2g} + z_2 \]

For turbulent flow set \( \alpha_1 = \alpha_2 \approx 1 \).

The faucet is a free jet, so \( p_2 = 0 \) (gauge)

Set \( z_1 = 0.0 \text{ m} \) and \( z_2 = 6.10 \text{ m} \)

The exit velocity \( v_2 = \frac{Q}{A_2} = 5.98 \text{ m/s} \)
Type I example: No head loss

Set $h_L = 0.0 \text{ m}$. So energy equation gives

$$\frac{p_1}{\gamma} = \frac{p_2}{\gamma} + \frac{v_2^2 - v_1^2}{2g} + (z_2 - z_1)$$

$$\frac{p_1}{9800} = 0 + \frac{5.98^2 - 2.67^2}{19.6} + (6.10)$$

$$\frac{p_1}{9800} = 7.56$$

$$\Rightarrow p_1 = 74.1 \text{ kPa}$$

About 80% of the pressure drop (from $z_1 \rightarrow z_2$) is due to the elevation increase while 20% is due to the velocity increase.
Type I example: Major head loss

The total length of the copper pipe is 18.3 m (60 ft).
Copper pipes are drawn tubing so $\varepsilon = 0.0015 \text{ mm}$.
Therefore $\varepsilon/D = 7.9 \times 10^{-5}$. Friction factor from Moody is 0.0215 (It is practically smooth).
So head loss is

$$h_{L\text{major}} = \frac{fLV^2}{2Dg}$$

$$= \frac{0.0215 \times 18.3 \times 2.67^2}{2 \times 0.019 \times 9.80}$$

$$= 7.53 \text{ m}$$

Simply add this head loss to

$$\frac{p_1}{\gamma} = 7.56 + h_{L\text{major}} = 7.56 + 7.53$$

$$\Rightarrow p_1 = 15.09 \times 9.8 = 148 \text{ kPa}$$
Type I example: Minor head loss

Now include impact of bends and valves. There are 4 $90^\circ$ elbows, each with $K_L = 1.5$.
The open globe valve has $K_L = 10$.
The loss coefficient of the faucet is $K_L = 2$.
To get the minor head losses, one simply adds up the individual losses from each component

$$h_{L\text{minor}} = \sum_i h_{L\text{minor}_i}$$

$$h_{L\text{minor}} = \sum_i K_{Li} \frac{v^2}{2g}$$

$$h_{L\text{minor}} = (10 + 4 \times 1.5 + 2) \frac{v^2}{2g} = 18.0 \frac{v^2}{2g}$$

$$h_{L\text{minor}} = 18.0 \frac{2.67^2}{19.6} = 6.55 \text{ m}$$

So the total head loss is $7.56 + 7.53 + 6.55 = 21.64$ m. The corresponds to a pressure difference of $21.64 \times 9.8 = 212.1 \text{ kPa}$. The gauge pressure $p_1 = 212.1 \text{ kPa}$.
Solution

\[ V_1 = \frac{Q}{A_1} = ... = 8.70 \text{ft/s} \]
\[ \mu = 2.34 \times 10^{-5} \text{lb} \cdot \text{s/ft}^2 \]
\[ \rho = 1.94 \text{slug/ft}^3 \]
\[ \text{Re} = \rho V D / \mu = 45000 \]

The flow is turbulent

The energy equation

\[ \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L \]

\[ p_1 = \gamma z_2 + \frac{1}{2} \rho (V_2^2 - V_1^2) + \gamma h_L \]

\[ z_1 = 0, z_2 = 20 \text{ft}, p_2 = 0 (\text{free jet}) \]

\[ V_2 = Q / A_2 = ... = 19.6 \text{ft/s} \]

Head loss is different for each of the three cases.
Solution $^{2/4}$

(a) If all losses are neglected ($h_L = 0$)

\[ p_1 = \gamma z_2 + \frac{1}{2} \rho (V_2^2 - V_1^2) = \ldots = 1547 \text{lb/ft}^2 = 10.7 \text{psi} \]

(b) If the only losses included are the major losses, the head loss is

\[ h_L = f \frac{\ell V_1^2}{D 2g} \]

\[ \varepsilon = 0.000005 \quad \varepsilon / D = 8 \times 10^{-5} \quad \text{Re} = 45000 \]

\[ f = 0.0215 \]

\[ p_1 = \gamma z_2 + \frac{1}{2} \rho (V_2^2 - V_1^2) + 4 \rho f \frac{\ell (= 60 \text{ft}) V_1^2}{D 2} = \ldots = 3062 \text{lb/ft}^2 = 21.3 \text{psi} \]
(c) If major and minor losses are included

\[ p_1 = \gamma z_2 + \frac{1}{2} \rho (V_2^2 - V_1^2) + f_\gamma \frac{\ell}{D} \frac{V_1^2}{2g} + \sum \rho K_L \frac{V^2}{2} \]

\[ p_1 = 21.3 \text{ psi} + \sum \rho K_L \frac{V^2}{2} \]

\[ = 21.3 \text{ psi} + (1.94 \text{ slugs/ft}^3) \frac{(8.70 \text{ ft/s})^2}{2} [10 + 4(1.5) + 2] \]

\[ p_1 = 21.3 \text{ psi} + 9.17 \text{ psi} = 30.5 \text{ psi} \]
Type III with minor losses. Example

Water at 10 °C with kinematic viscosity

\[ 1.31 \times 10^{-6} \text{ m}^2/\text{s} \]

is to flow from \( A \) to \( B \) through a cast-iron pipe \( \varepsilon = 0.26 \text{ mm} \) at a rate of \( 0.0020 \text{ m}^3/\text{s} \).

The system contains a sharp-edged entrance and six threaded 90° elbows. Determine the pipe diameter that is needed.

Will use the energy equation

\[
\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_L
\]

With reference points at (1) and (2) so that \( p_1 = p_2 = 0 \), \( z_2 = 0 \) and \( v_1 = v_2 = 0 \).
Type III with minor losses. Example

The energy equation simplifies

\[
\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_L
\]

\[
z_1 = h_L
\]

\[
z_1 = \frac{v^2}{2g} \left( \frac{f \ell}{D} + \sum_i K_{Li} \right)
\]

where \( v \) is water velocity in pipe.

\[
v = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{2.55 \times 10^{-3}}{D^2}
\]
Type III with minor losses. Example

Head-loss terms

- Six 90° elbows. $K_L = 6 \times 1.5 = 9.0$
- Tank → Pipe. $K_L = 0.5$
- Pipe → Tank. $K_L = 1.0$
- Total: $\sum K_L = 10.5$

The energy equation becomes

$$z_1 = \frac{v^2}{2g} \left( \frac{f \ell}{D} + \sum_i K_{Li} \right)$$

$$2.0 = \frac{6.50 \times 10^{-6}}{2 \times 9.80 \times D^2} \left( \frac{f \times 20}{D} + 10.5 \right)$$

The problem with this equation is that the friction factor depends on $D$ in a complicated manner. The friction factor depends on the roughness $\varepsilon/D$ and Reynolds number $\rho v D/\mu$.

What we want to do is choose $D$ so that the head loss is 2.0 m. This is a non-linear equation.
Solving the pipe sizing problem

Want to solve

\[ 2.0 = \frac{6.50 \times 10^{-6}}{2 \times 9.80 \times D^2} \left( \frac{f \times 20}{D} + 10.5 \right) \]

The head-loss (right-hand side) will get smaller as \( D \) gets larger. Procedure

- Need to bootstrap the problem (supply initial guess). Set \( f = 0.0200 \) (a reasonable value for many pipe problems) get an initial value of \( D \), namely \( D_0 \).
- Plug initial estimate of \( D_0 \) into \( RHS \). Use \( D_0 \rightarrow \varepsilon/D \rightarrow f \rightarrow h_L \). Use Moody diagram or approximate formula for specific flow regime.
- If \( h_L < 2.0 \), then \( D \) needs to be decreased. If \( h_L > 2.0 \), then \( D \) needs to be increased. Use \( h_L \propto 1/D^2 \) scaling to get next estimate.
- Keep record of \( h_L \) vs \( D \). When have enough points, plot \( h_L \) vs \( D \) and determine where \( h_L = 2.0 \text{ m} \) is true. Note, no point in getting \( D \) to better than 1% accuracy.
Solving the pipe sizing problem

The actual solution occurs when self-consistency occurs.

Note, pipes come with certain standard diameters. Choose a pipe diameter that is larger than exact diameter extracted from equation.
Minor Losses

- One source of head-loss occurs when the pipe diameters change.
- These changes can be abrupt or smooth.
- One of the reasons for head loss is that it is not possible to slow down a fluid easily.
Entrance flow condition and loss coefficient

(a) re-entrant, \( K_L = 0.8 \)
(b) sharp-edged, \( K_L = 0.5 \)
(c) slightly rounded, \( K_L = 0.2 \)
(d) well-rounded, \( K_L = 0.04 \)
Minor Losses

- About 50% of the energy is lost when the fluid enters a pipe with a square edged entrance.
- Rounding the entrance corner will reduce the loss coefficient.
- If the pipe protrudes into the tank the loss coefficient will be even larger.
Minor Losses

- The head loss when water from a large pipe enters a tank is $KL = 1$ irrespective of the geometry.
- The fluid from the pipe mixes with the fluid and its kinetic energy is dissipated through viscous effects as the fluid eventually comes to rest.
As a fluid flows from a smaller pipe into a larger pipe through a sudden enlargement, its velocity abruptly decreases, causing turbulence, which generates an energy loss.

Figure below shows the sudden enlargement.
The minor loss is calculated from the equation

\[ h_L = K\left(\frac{v_1^2}{2g}\right) \]

where \( v_1 \) is the average velocity of flow in the smaller pipe ahead of the enlargement.

By making some simplifying assumptions about the character of the flow stream as it expands through the sudden enlargement, it is possible to analytically predict the value of \( K \) from the following equation:

\[ K = \left[1 - \left(\frac{A_1}{A_2}\right)^2\right]^2 = \left[1 - \left(\frac{D_1}{D_2}\right)^2\right]^2 \]
SUDDEN ENLARGEMENT

Fig below shows the resistance coefficient—sudden enlargement.
EXAMPLE – SUDDEN ENLARGEMENT

Determine the energy loss that will occur as 100 L/min of water flows through a sudden enlargement from a 1-in copper tube (Type K) to a 3-in tube (Type K). See Appendix H for tube dimensions.
EXAMPLE 5 – SUDDEN ENLARGEMENT

Using the subscript 1 for the section just ahead of the enlargement and 2 for the section downstream from the enlargement, we get

\[
D_1 = 25.3 \text{ mm} = 0.0253 \text{ m} \\
A_1 = 5.017 \times 10^{-4} \text{ m}^2 \\
D_2 = 73.8 \text{ mm} = 0.0738 \text{ m} \\
A_2 = 4.282 \times 10^{-3} \text{ m}^2 \\
\]

\[
v_1 = \frac{Q}{A_1} = \frac{100 \text{ L/min}}{5.017 \times 10^{-4} \text{ m}^2} \times \frac{1 \text{ m}^3/\text{s}}{60000 \text{ L/min}} = 3.32 \text{ m/s} \\
v_1^2 = \frac{(3.32)^2}{2g} \text{ m} = 0.56 \text{ m}
\]
EXAMPLE 5 – SUDDEN ENLARGEMENT

To find a value for $K$, the diameter ratio is needed. We find that

$$\frac{D_2}{D_1} = \frac{73.8}{25.3} = 2.92$$

Try to obtained from graph( Resistance coefficient – Sudden enlargement ), $K = 0.72$. Then we have

$$h_L = K\left(\frac{v_1^2}{2g}\right) = (0.72)(0.56 \text{ m}) = 0.40 \text{ m}$$

This result indicates that 0.40 Nm of energy is dissipated from each Newton of water that flows through the sudden enlargement.
EXAMPLE – SUDDEN ENLARGEMENT

Determine the difference between the pressure ahead of a sudden enlargement and the pressure downstream from the enlargement. Use the data from Example 5.

First, we write the energy equation:

\[
\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g}
\]

\[
p_1 - p_2 = \gamma[(z_2 - z_1) + (v_2^2 - v_1^2)/2g + h_L]
\]
EXAMPLE — SUDDEN ENLARGEMENT

- If the enlargement is horizontal, \( z_2 - z_1 = 0 \).
- Even if it were vertical, the distance between points 1 and 2 is typically so small that it is considered negligible.
- Now, calculating the velocity in the larger pipe, we get

\[
v_2 = \frac{Q}{A_2} = \frac{100 \text{ L/min}}{4.282 \times 10^{-3} \text{ m}^2} \times \frac{1 \text{ m}^3/\text{s}}{60 000 \text{ L/min}} = 0.39 \text{ m/s}
\]
EXAMPLE – SUDDEN ENLARGEMENT

Using \( \gamma = 9.81 \text{ kN/m}^3 \) for water and \( h_L = 0.40 \text{m} \) from Example Problem 10.1, we have

\[
p_1 - p_2 = \frac{9.81 \text{ kN}}{\text{m}^3} \left[ 0 + \frac{(0.39)^2 - (3.32)^2}{(2)(9.81)} \text{m} + 0.40 \text{m} \right]
\]

\[
= -1.51 \text{ kN/m}^2 = -1.51 \text{ kPa}
\]

Therefore, \( p_2 \) is 1.51 kPa greater than \( p_1 \).
ENERGY LOST IN GRADUAL ENLARGEMENT

- If the transition from a smaller to a larger pipe can be made less abrupt than the square-edged sudden enlargement, the energy loss is reduced.
- This is normally done by placing a conical section between the two pipes as shown in the below figure.
ENERGY LOST IN GRADUAL ENLARGEMENT

Compare gradual enlargement (left) to sudden enlargement (right).
ENERGY LOST IN GRADUAL ENLARGEMENT

The energy loss for a gradual enlargement is calculated from

\[ h_L = K\left(\frac{v_1^2}{2g}\right) \]

Data for various values are given below

<table>
<thead>
<tr>
<th>( D_2/D_1 )</th>
<th>(2^\circ)</th>
<th>(6^\circ)</th>
<th>(10^\circ)</th>
<th>(15^\circ)</th>
<th>(20^\circ)</th>
<th>(25^\circ)</th>
<th>(30^\circ)</th>
<th>(35^\circ)</th>
<th>(40^\circ)</th>
<th>(45^\circ)</th>
<th>(50^\circ)</th>
<th>(60^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
<td>0.10</td>
<td>0.13</td>
<td>0.16</td>
<td>0.18</td>
<td>0.19</td>
<td>0.20</td>
<td>0.21</td>
<td>0.23</td>
</tr>
<tr>
<td>1.2</td>
<td>0.02</td>
<td>0.02</td>
<td>0.04</td>
<td>0.09</td>
<td>0.16</td>
<td>0.21</td>
<td>0.25</td>
<td>0.29</td>
<td>0.31</td>
<td>0.33</td>
<td>0.35</td>
<td>0.37</td>
</tr>
<tr>
<td>1.4</td>
<td>0.02</td>
<td>0.03</td>
<td>0.06</td>
<td>0.12</td>
<td>0.23</td>
<td>0.30</td>
<td>0.36</td>
<td>0.41</td>
<td>0.44</td>
<td>0.47</td>
<td>0.50</td>
<td>0.53</td>
</tr>
<tr>
<td>1.6</td>
<td>0.03</td>
<td>0.04</td>
<td>0.07</td>
<td>0.14</td>
<td>0.26</td>
<td>0.35</td>
<td>0.42</td>
<td>0.47</td>
<td>0.51</td>
<td>0.54</td>
<td>0.57</td>
<td>0.61</td>
</tr>
<tr>
<td>1.8</td>
<td>0.03</td>
<td>0.04</td>
<td>0.07</td>
<td>0.15</td>
<td>0.28</td>
<td>0.37</td>
<td>0.44</td>
<td>0.50</td>
<td>0.54</td>
<td>0.58</td>
<td>0.61</td>
<td>0.65</td>
</tr>
<tr>
<td>2.0</td>
<td>0.03</td>
<td>0.04</td>
<td>0.07</td>
<td>0.16</td>
<td>0.29</td>
<td>0.38</td>
<td>0.46</td>
<td>0.52</td>
<td>0.56</td>
<td>0.60</td>
<td>0.63</td>
<td>0.68</td>
</tr>
<tr>
<td>2.5</td>
<td>0.03</td>
<td>0.04</td>
<td>0.08</td>
<td>0.16</td>
<td>0.30</td>
<td>0.39</td>
<td>0.48</td>
<td>0.54</td>
<td>0.58</td>
<td>0.62</td>
<td>0.65</td>
<td>0.70</td>
</tr>
<tr>
<td>3.0</td>
<td>0.03</td>
<td>0.04</td>
<td>0.08</td>
<td>0.16</td>
<td>0.31</td>
<td>0.40</td>
<td>0.48</td>
<td>0.55</td>
<td>0.59</td>
<td>0.63</td>
<td>0.66</td>
<td>0.71</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0.03</td>
<td>0.05</td>
<td>0.08</td>
<td>0.16</td>
<td>0.31</td>
<td>0.40</td>
<td>0.49</td>
<td>0.56</td>
<td>0.60</td>
<td>0.64</td>
<td>0.67</td>
<td>0.72</td>
</tr>
</tbody>
</table>

The energy loss calculated from previous does not include the loss due to friction at the walls of the transition.

For relatively steep cone angles, the length of the transition is short and therefore the wall friction loss is negligible.
Determine the energy loss that will occur as 100 L/min of water flows from a 1-in copper tube (Type K) into a 3-in copper tube (Type K) through a gradual enlargement having an included cone angle of 30 degrees.

\[
v_1 = 3.32 \text{ m/s}
\]

\[
v_1^2/2g = 0.56 \text{ m}
\]

\[
D_2/D_1 = 73.8/25.3 = 2.92
\]
EXAMPLE - GRADUAL ENLARGEMENT

From Graph (Resistance coefficient – gradual enlargement), we find that $K = 0.48$. Then we have

$$h_L = K\left(\frac{v_1^2}{2g}\right) = (0.48)(0.56 \text{ m}) = 0.27 \text{ m}$$

Compared with the sudden enlargement described in Example 5, the energy loss decreases by 33% when 30 degrees the gradual enlargement is used.
The energy loss due to a sudden contraction, such as that sketched in Fig. 10.6, is calculated from:

\[ h_L = K\left(\frac{v_2^2}{2g}\right) \]

where \( v^2 \) is the velocity in the small pipe downstream from the contraction.

Figure 10.8 illustrates what happens as the flow stream converges.

The lines in the figure represent the paths of various parts of the flow stream called *streamlines*. 
RESISTANCE COEFFICIENT - SUDDEN CONTRACTION
SUDDEN CONTRACTION
EXAMPLE 8 – SUDDEN CONTRACTION

Determine the energy loss that will occur as 100 L/min of water flows from a 3-in copper tube (Type K) into a 1-in copper tube (Type K) through a sudden contraction.

Head lost is

\[ h_L = K \left( \frac{v_2^2}{2g} \right) \]

For the copper tube,

\[
\frac{D_1}{D_2} = \frac{73.8}{25.3} = 2.92
\]

\[
v_2 = \frac{Q}{A_2} = \frac{100 \text{ L/min}}{5.017 \times 10^{-4} \text{ m}^2} \times \frac{1 \text{ m}^3/\text{s}}{60000 \text{ L/min}} = 3.32 \text{ m/s}
\]

\[
\frac{v_2^2}{2g} = 0.56 \text{ m}
\]
EXAMPLE 8 – SUDDEN CONTRACTION

From graph, $K = 0.42$. Then we have

$$h_L = K\left(\frac{v_2^2}{2g}\right) = (0.42)(0.56 \text{ m}) = 0.24 \text{ m}$$
The following figure shows that the minimum resistance for a $90^\circ$ bend occurs when the ratio $r/D$ is approximately three.
The following figure shows a 90° bend.
The following figure shows a $90^\circ$ bend pipe.

If $R_o$ is the radius to the outside of the bend, $R_i$ is the radius to the inside of the bend and $D_o$ is the outside diameter of the pipe or tube. The radius to the centerline of the pipe or tube called mean radius, $r$ can be expressed as:

\[
\begin{align*}
    r &= R_i + \frac{D_o}{2} \\
    r &= R_o - \frac{D_o}{2} \\
    r &= \frac{(R_o + R_i)}{2}
\end{align*}
\]
A distribution system for liquid propane is made from 1.25 in drawn steel tubing with a wall thickness of 0.083 in. Several 90° bends are required to fit the tubes to the other equipment in the system. The specifications call for the radius to the inside of each bend to be 200 mm. When the system carries 160 L/min of propane at 25°C, compute the energy loss to each bend.
The radius \( r \) must be computed from

\[ r = R_i + D_o/2 \]

where \( D_o = 31.75 \text{mm} \), the outside diameter of the tube as found from Appendix G.

Completion of the calculation gives

\[ r = 200 \text{mm} + (31.75 \text{mm})/2 = 215.9 \text{mm} \]

D (internal diameter) = \( D_o - 2 \times \text{wall thickness} \)

\[ r/D = 215.9 \text{mm}/27.5 \text{mm} = 7.85 \]
We now must compute the velocity to complete the evaluation of the energy loss from Darcy’s equation:

\[
v = \frac{Q}{A} = \frac{160 \text{ L/min}}{5.954 \times 10^{-4} \text{ m}^2} \times \frac{1.0 \text{ m}^3/\text{s}}{60000 \text{ L/min}} = 4.48 \text{ m/s}
\]

The relative roughness is

\[
\frac{D}{\epsilon} = \frac{0.0275 \text{ m}}{1.5 \times 10^{-6} \text{ m}} = 18333
\]
EXAMPLE - PIPE BENDS

Then, we can find $f_T = 0.0108$ from the Moody diagram (Fig. 8.6) in the zone of complete turbulence. Then

$$K = f_T \left( \frac{L_e}{D} \right) = 0.0108(23) = 0.248$$

Now the energy loss can be computed:

$$h_L = K \frac{v^2}{2g} = 0.248 \frac{(4.48)^2}{(2)(9.81)} = 0.254 \text{ m} = 0.254 \text{ N·m/N}$$