Influence Lines for Statically Determinate Structures
Definition:

An influence line represents the variation of either the reaction, shear, moment, or deflection at a specific point in a member as a concentrated force moves over the member. Once this line is constructed, one can tell at a glance where the moving load should be placed on the structure so that it creates the greatest influence at the specified point.
The main difference between constructing an influence line and constructing a shear or moment diagram is:
Influence lines represent the effect of a moving load only at a specified point on a member, whereas shear and moment diagrams represent the effect of fixed loads at all points along the axis of the member.
1. Influence Lines for Trusses

An influence line for truss member can be constructed by loading each joint along the deck with a unit load and then use the method of joints or the method of sections to calculate the force in the member. The data can be arranged in tabular form, listing “unit load at joint” versus “force in member.”
Example:

Draw the influence line for the force in member \textit{GB} of the bridge truss shown in Fig. below.

![Bridge Truss Diagram]
Solution:

At \( X = 6 \text{m} \)

Similarly:

\[
\Sigma F_y = 0; \quad 0.25 - F_{GB} \sin 45^\circ = 0
\]

\[
F_{GB} = 0.354
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( F_{GB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.354</td>
</tr>
<tr>
<td>12</td>
<td>-0.707</td>
</tr>
<tr>
<td>18</td>
<td>-0.354</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
</tr>
</tbody>
</table>
The point of zero force, is determined by similar triangles between $x=6\text{m}$ and $x=8\text{m}$ that is,

$$(0.354 + 0.7072)/(12 - 6) = 0.354/x'$$

$x' = 2\text{m}$, so $x = 6 + 2 = 8\text{m}$. 
Example:

Draw the influence line for the force in member $CG$ of the bridge truss shown in Fig. below.
Solution:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$F_{GC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
</tr>
</tbody>
</table>

![Diagram of influence line for $F_{CG}$]

Influence line for $F_{CG}$
Influence Lines for Beams
Two types of loadings will now be considered.

Concentrated Force: Since the numerical values of a function for an influence line are determined using a dimensionless unit load, then for any concentrated force $F$ acting on the beam at any position $x$, the value of the function can be found by multiplying the ordinate of the influence line at the position $x$ by the magnitude of $F$.

$$A_y = \left(\frac{1}{2}\right)(F)$$
\[ dF = w_0 \, dx \]

influence line for function
Example:

Construct the influence line for
a) reaction at A and B
b) shear at point C
c) bending moment at point C
d) shear before and after support B
e) moment at point B
of the beam in the figure below.
Solution:

- Reaction at A

<table>
<thead>
<tr>
<th>x</th>
<th>A_y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

\[ \Sigma M_B = 0: \quad -A_y(8) + 1(8 - x) = 0, \quad A_y = 1 - \frac{1}{8}x \]
• Reaction at B

<table>
<thead>
<tr>
<th>$x$</th>
<th>$B_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>1.5</td>
</tr>
</tbody>
</table>

$$\Sigma M_A = 0: \quad B_y(8) - 1x = 0,$$

$$B_y = \frac{1}{8}x$$
**Shear at C**

\[ 0 \leq x < 4 \quad 4 < x \leq 12 \]

\[ \begin{align*}
  A & \quad \Rightarrow \quad A_y = 1 - \frac{1}{8}x \\
  C & \quad \Rightarrow \quad V_C = \frac{1}{8}x \\
  B & \quad \Rightarrow \quad B_y
\end{align*} \]

**Case 1: 0 \leq x \leq 4**

\[ +\Sigma F_y = 0: \quad 1 - \frac{1}{8}x - 1 - V_C = 0 \]
\[ V_C = -\frac{1}{8}x \]

**Case 2: 4 < x \leq 12**

\[ +\Sigma F_y = 0: \quad 1 - \frac{1}{8}x - V_C = 0 \]
\[ V_C = 1 - \frac{1}{8}x \]
\[ V_c = -\frac{1}{8}x \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( V_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4^-</td>
<td>-0.5</td>
</tr>
<tr>
<td>4^+</td>
<td>0.5</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>-0.5</td>
</tr>
</tbody>
</table>
• Bending moment at C

\[0 \leq x < 4\]
\[4 < x \leq 12\]

\[\begin{align*}
A_x &= 1 - \frac{1}{8}x \\
M_C &= 1 \sum M_C = 0: \quad M_C + 1(4 - x) - (1 - \frac{1}{8}x)(4) = 0 \\
    &= M_C = \frac{1}{2}x \\
\end{align*}\]

\[\begin{align*}
A_x &= 1 - \frac{1}{8}x \\
M_C &= \sum M_C = 0: \quad M_C - (1 - \frac{1}{8}x)(4) = 0 \\
    &= M_C = 4 - \frac{1}{2}x \\
\end{align*}\]

Dr. Nasrellah H A
\[ M_C = \frac{1}{2} x \]

\[ M_C = 4 - \frac{1}{2} x \]

\[
\begin{array}{|c|c|}
\hline
x & M_C \\
\hline
0 & 0 \\
4 & 2 \\
8 & 0 \\
12 & -2 \\
\hline
\end{array}
\]
• Shear before support B

\[ V_B^- = A_y - 1 \]

\[ V_B^- = A_y \]
Shear after support B

\[ V_B^+ = 0 \]

\[ V_B^+ = 1 \]
• Moment at support B

\[ M_B = 8A_y - (8-x) \]
Example:

Determine the maximum reaction at support B, the maximum shear at point C and the maximum *positive* moment that can be developed at point C on the beam shown due to

- a single concentrate live load of 8000 N
- a uniform live load of 3000 N/m
- a beam weight (dead load) of 1000 N/m
SOLUTION

\[ (R_B)_{\text{max}} = (1000)(9) + (3000)(9) + (8000)(1.5) \]

\[ = 48000 \text{ N} = 48 \text{ kN} \]
\[ V_C \]

\[ (V_C)_{\text{max}} = (1000)(-2+1) + (3000)(-2) + (8000)(-0.5) \]

\[ = -11000 \text{ N} = 11 \text{ kN} \]
The bending moment diagram for the given beam can be calculated as follows:

\[ (M_C)_{\text{max positive}} = (8000)(2) + (3000)(8) + (8-4)(1000) \]

\[ = 44000 \text{ N} \cdot \text{m} = 44 \text{ kN} \cdot \text{m} \]
Absolute Maximum Shear and Moment

\[ \Sigma M_B = 0; \quad A_y = \frac{1}{L} (F_R) \left[ \frac{L}{2} - (\bar{x}' - x) \right] \]
\[ \Sigma M = 0; \quad M_2 = A_y \left( \frac{L}{2} - x \right) - F_1d_1 \]
\[ = \frac{1}{L} (F_R) \left[ \frac{L}{2} - (\bar{x}' - x) \right] \left( \frac{L}{2} - x \right) - F_1d_1 \]
\[ = \frac{F_R L}{4} - \frac{F_R x'}{2} - \frac{F_R x^2}{L} + \frac{F_R x x'}{L} - F_1d_1 \]

For maximum \( M_2 \) we require
\[ \frac{dM_2}{dx} = \frac{-2F_R x}{L} + \frac{F_R x'}{L} = 0 \]
or
\[ x = \frac{\bar{x}'}{2} \]

Hence, we may conclude that the absolute maximum moment in a simply supported beam occurs under one of the concentrated forces, such that this force is positioned on the beam so that it and the resultant force of the system are equidistant from the beam’s centerline.
Determine the absolute maximum moment in the simply supported bridge deck shown in Fig. 6–37a.
SOLUTION

The magnitude and position of the resultant force of the system are determined first, Fig. 6–37a. We have

\[ + \downarrow F_R = \Sigma F; \quad F_R = 2 + 1.5 + 1 = 4.5 \text{ k} \]

\[ \uparrow + M_{RC} = \Sigma M_C; \quad 4.5\bar{x} = 1.5(10) + 1(15) \]

\[ \bar{x} = 6.67 \text{ ft} \]
\( \sum M_B = 0; \quad -A_y(30) + 4.5(16.67) = 0 \quad A_y = 2.50 \text{k} \)

Now using the left section of the beam, Fig. 6–37c, yields

\( \sum M_S = 0; \quad -2.50(16.67) + 2(10) + M_S = 0 \)

\[ M_S = 21.7 \text{k⋅ft} \]

\[ A_y = 2.5 \text{k} \]