Chapter 12

Dynamics of closed loop
Components and Signals of a Typical Control Loop

- **Components:**
  - Thermocouple
  - Transmitter
  - 4-20 ma DC
  - DCS Control Computer
  - A/D
  - Thermowell
  - Operator Console

- **Signals:**
  - Air
  - 3-15 psig
  - 4-20 ma
  - T
  - F
  - I/P
  - D/A
  - A/D
  - Transmitter
  - Tsp

- **Diagram Notes:**
  - F1, T1, F2, T2
  - Thermocouple millivolt signal
  - Thermowell
  - 4-20 ma signal transmits from A/D to DCS Control Computer.
Types of Control

- **manual**
  - no automatic adjustment
  - human operator provides feed back
  - Wood stove example

- **automatic controls - using a variety of algorithms**
  - on/off - will result in continuous cycling
    (think of your house heating, or your oven)
  - PID and other control algorithms

- **emergency control**
  - alarms, pressure venting
  - sequence of shutdown steps
Controllers/Control Computers

- Pneumatic controllers
- Electronic analog controllers
- Supervisory control computers
- Distributed Control Systems (DCS)
- Fieldbus technology
Pneumatic Controller Installation

- Thermocouple millivolt signal
- Transmitter
- Pneumatic Controller
- 3-15 psig Air
- Thermowell
- Thermowell millivolt signal
- Pneumatic Controller
- 3-15 psig Air
Electronic Controller Installation

Electronic Analog Controller

3-15 psig

4-20 ma

T

I/P

Air

T

F1

F2

T1

T2

Thermowell

Transmitter

Thermocouple millivolt signal
Computer Control System

- Based upon a mainframe digital computer.
- Offered the ability to use data storage and retrieval, alarm functions, and process optimization.
- First installed on a refinery in 1959.
- Had reliability limitations.
Supervisory Control Computer

Video Display Unit

Alarming Functions

Printer

Supervisory Control Computer

Analog Control Subsystem

Interfacing Hardware

Data Storage Acquisition System
DCS Architecture

System Consoles

Host Computer

Data Storage Unit

PLC

Data Highway
(Shared Communication Facilities)

Local Console

Local Control Unit

Local Control Unit

Local Console

Process Transmitters and Actuators
Fieldbus Technology

- Based upon **smart valves, smart sensors** and controllers installed in the field.
- Uses data highway to replace wires from sensor to DCS and to the control valves.
- **Less expensive installations** and better reliability.
- **Can mix different sources** (vendors) of sensors, transmitters, and control valves.
- Now commercially available and should begin to replace DCSs.
Fieldbus Architecture

Plant-Wide Network

Local Area Network

Smart Sensors
Smart Control
Valves and Controllers

Local Area Network

Smart Sensors
Smart Control
Valves and Controllers

Fieldbus Network
Fieldbus Network
Basic Control Modes

- Proportional Control, (P)
- Integral Control; Proportional-Integral (PI) controller
- Derivative Control; Proportional-Derivative (PD) controller
- Proportional-Integral-Derivative Controller (PID)
Proportional Control

In feedback control, the objective is to reduce the error signal to zero where

\[ e(t) = y_{sp}(t) - y_{m}(t) \]  \hspace{1cm} (8-1)

and

\[ e(t) \text{ = error signal} \]
\[ y_{sp}(t) \text{ = set point} \]
\[ y_{m}(t) \text{ = measured value of the controlled variable} \]
\[ \text{(or equivalent signal from the sensor/transmitter)} \]
Although Eq. 8-1 indicates that the set point can be time-varying, in many process control problems it is kept constant for long periods of time.

For proportional control, the controller output is proportional to the error signal,

\[ p(t) = \bar{p} + K_c e(t) \]  \hspace{1cm} (8-2)

where:

\( p(t) \) = controller output
\( \bar{p} \) = bias (steady-state) value
\( K_c \) = controller gain (usually dimensionless)
Figure 8.4. Proportional control: ideal behavior (slope of line = $K_c$).

Figure 8.5. Proportional control: actual behavior.
In order to derive the transfer function for an ideal proportional controller (without saturation limits), define a deviation variable $p'(t)$ as

$$p'(t) = p(t) - \bar{p} \quad (8-4)$$

Then Eq. 8-2 can be written as

$$p'(t) = K_c e(t) \quad (8-5)$$

The transfer function for proportional-only control:

$$\frac{P'(s)}{E(s)} = K_c \quad (8-6)$$

An inherent disadvantage of proportional-only control is that a steady-state error occurs after a set-point change or a sustained disturbance.
The key concepts behind proportional control are the following:

1. The controller gain can be adjusted to make the controller output changes as sensitive as desired to deviations between set point and controlled variable;

2. the sign of $K_c$ can be chose to make the controller output increase (or decrease) as the error signal increases.

For proportional controllers, bias $p$ can be adjusted, a procedure referred to as *manual reset*.

Some controllers have a proportional band setting instead of a controller gain. The *proportional band* $PB$ (in %) is defined as

$$PB = \frac{100\%}{K} \quad (8-3)$$
Proportional Band

\[ PB = \frac{100\%}{K_c} \]

- Another way to express the controller gain.
- \( K_c \) in this formula is dimensionless. That is, the controller output is scaled 0-100\% and the error from setpoint is scaled 0-100\%.
- In more frequent use 10-15 years ago, but it still appears as an option on DCS’s.
The Deficiency of Proportional Control

- making an adjustment to the MV that is proportional to the error is reasonable *BUT*, if we still have an error when we get to steady state, the controller won’t take any further action.

- leads to sustained deviation - *offset*

- proportional controllers make adjustments based on what is happening in the *present*

\[
MV = K_c (SP - CV) + bias = K_c e(t) + bias
\]
Proportional Feedback

Example:
Given first order process:

\[ G_p(s) = \frac{K_p}{\tau s + 1}, \quad G_v(s) = 1, \quad G_m(s) = 1 \]

for P-only feedback closed-loop dynamics:

\[ Y(s) = \frac{K_pK_c}{1 + K_pK_c} R(s) - \frac{\tau}{1 + K_pK_c} s + 1 \]

\[ + \left( \frac{\tau}{1 + K_pK_c} s + 1 \right) \frac{1}{1 + K_pK_c} D(s) \]

Closed-Loop Time Constant
Proportional Feedback

Final response:

\[
\lim_{t \to \infty} y_{\text{servo}}(t) = \frac{K_p K_c}{1 + K_p K_c}, \quad \lim_{t \to \infty} y_{\text{reg}}(t) = \frac{1}{1 + K_p K_c}
\]

Note:

- for “zero offset response” we require

\[
\lim_{t \to \infty} y_{\text{servo}}(t) = 1, \quad \lim_{t \to \infty} y_{\text{reg}}(t) = 0
\]

- Possible to eliminate offset with P-only feedback (requires infinite controller gain)

- Need different control action to eliminate offset (integral)

Tracking Error

Disturbance rejection
Proportional Feedback

Servo dynamics of a first order process under proportional feedback

- increasing controller gain eliminates off-set
Proportional Feedback

High-order process

e.g. second order underdamped process, two 1st orders, 1st order process and 1st order measurement.

Increasing controller gain reduces offset, speeds response and increases oscillation.
Proportional Feedback

Important points:

- Proportional feedback does not change the order of the system
  - Started with a first order process
  - Closed-loop process also first order
  - Order of characteristic polynomial is invariant under proportional feedback

- Speed of response of closed-loop process is directly affected by controller gain
  - Increasing controller gain reduces the closed-loop time constant

- In general, proportional feedback
  - Reduces (does not eliminate) offset
  - Speeds up response
  - For oscillatory processes, makes closed-loop process more oscillatory
See example 11.2, 11.3
8.3.2.2 Roles of Three Parts

- **Proportional part**: Since the control output $u_P(t)$ is proportional to the error $(y_s(t) - y(t))$, it plays a role in pushing the process output to the **set point** as much as the error.

1. Transfer function.

$$u_P(t) = k_c (y_s(t) - y(t)) = k_c e(t) \quad \Rightarrow \quad \frac{U_P(s)}{E(s)} = k_c \quad (8.5)$$


3. Disadvantage: **steady-state error (offset)**.

4. Usage: when the steady-state error is tolerable (ex. level control which wants to prevent the system from overflowing or drying), proportional-only controller is attractive because of its simplicity, seldom used only.

To remove the steady-state error (offset), the **integral control action** should be included in the feedback controller.
Steady-state error.

For usual process (i.e., open-loop stable processes), the control output should be nonzero to keep the process output in a nonzero set point.

- Consider the following PD controller (the following derivation is applicable to P controller case).

\[
    u_{PD}(t) = k_c (y_s(t) - y(t)) + k_c \tau_D \frac{d(y_s(t) - y(t))}{dt} \tag{8.6}
\]

PD (or P) controller output \( u_{PD}(t) \) is always zero at steady-state if the error is zero (i.e., \( y(t) = y_s(t) \)).

\( \Rightarrow \) PD (or P) controller cannot be nonzero constant when the error is zero at steady-state. So, the PD (or P) controller cannot keep the process output in a nonzero set point for open-loop stable processes.

- Offset can be calculated as follows. Here \( SS \) denotes steady state and \( k \) is the static gain (or DC gain) of the process.

\[
    y_{ss}(t) = k \cdot u_{ss}(t) = k \cdot k_c (y_s - y_{ss}(t)) \tag{8.7}
\]

\[
    \text{Offset: } y_s - y_{ss}(t) = \frac{y_s}{1 + k \cdot k_c} \tag{8.8}
\]
Integral part (= reset or floating control part): Since the integral part is not necessarily zero even though the error at steady-state is zero, it plays an important role in rejecting the offset.

\[ u_{PID,ss}(t) = \frac{k_c}{\tau_l} \int_0^t (y_s(t^*) - y(t^*)) dt^* = \text{nonzero constant} \quad (8.9) \]

1. Transfer function.
\[
\frac{U_I(s)}{E(s)} = \frac{k_c}{\tau_l} \cdot \frac{1}{s} \quad (8.10)
\]

2. Disadvantages

- Not immediate corrective action.
  - Practically PI controller is used.
- Oscillatory response.
  - Reduce the stability of the system.

Solution: proper tuning of the controller or including derivative control action which tends to counteract the destabilizing effects.

- Reset windup (or integral windup).
Reset windup (or Integral windup).

- Sustained error $\leftrightarrow$ Large integral term $\leftrightarrow$ Saturation of controller output $\leftrightarrow$ Further buildup of the integral term while the controller is saturated is referred to as reset windup or integral windup.
- Reset windup occurs when a PI or PID controller encounters a sustained error, for example, during the start-up of a batch process or after a large set-point change.

![Diagram showing reset windup during a set-point change.](image)

Figure 8.5. Reset windup during a set-point change.

- The large overshoot occurs because the integral term continues to increase until the error signal changes sign at $t = t_1$.
- Antireset windup; halting the integral action whenever the controller output saturates. Most of commercial controllers provides antireset windup.
**Derivative part (= rate action, pre-act or anticipatory control part)**

: Since this part represents approximately the increment of the error after time from the present time, it plays a role in rejecting the future error in advance by increasing the control output proportional to the future incremental error.

![Diagram of derivative part](image)

**1. Transfer function.**

\[
\frac{U_D(s)}{E(s)} = k_c \tau_D \cdot s \quad (8.11)
\]

**2. Advantage:** This part enhances the robustness of the PID controller by considering abrupt change of the error.

**3. Disadvantage:** If the process measurement is noisy, this term will change widely and amplify the noise unless the measurement is filtered.
Integral Control

Integrator is included to eliminate offset

- provides reset action
- usually added to a proportional controller to produce a PI controller
  - PID controller with derivative action turned off
  - PI is the most widely used controller in industry
  - optimal structure for first order processes

PI controller form

\[ u(t) = K_c \left[ e(t) + \frac{1}{\tau_I} \int_0^t e(\zeta) d\zeta \right] + u_R \]

Transfer function model

\[ U'(s) = K_c \left( 1 + \frac{1}{\tau_I s} \right) E(s) \]
Proportional-Integral (PI) Control

\[ p(t) = \bar{p} + K_c \left[ e(t) + \frac{1}{\tau_1} \int_0^t e(t')dt' \right] \]

- Response to unit step change in e:

![Graph showing the response of a proportional-integral controller](image)

Figure 8.6. Response of proportional-integral controller to unit step change in \( e(t) \).
PI Feedback

Closed-loop response

\[ Y(s) = \frac{G_p(s)G_v(s)K_c\left(\frac{\tau I s + 1}{\tau I s}\right)}{1 + G_p(s)G_v(s)K_c\left(\frac{\tau I s + 1}{\tau I s}\right)G_m(s)} R(s) + \frac{1}{1 + G_p(s)G_v(s)K_c\left(\frac{\tau I s + 1}{\tau I s}\right)G_m(s)} D(s) \]

- more complex expression
- degree of denominator is increased by one
Example

PI control of a first order process

\[ G_p(s) = \frac{K_p}{\tau s + 1}, \quad G_v(s) = 1, \quad G_m(s) = 1 \]

Closed-loop response

\[ Y(s) = \frac{\tau_I s + 1}{\left(\frac{\tau_I \tau}{K_c K_p}\right)s^2 + \left(\frac{1 + K_c K_p}{K_c K_p}\right)\tau_I s + 1} R(s) + \]

\[ \left(\frac{\tau_I \tau}{K_c K_p}\right)s^2 + \frac{\tau_I}{K_c K_p} s \]

\[ \left(\frac{\tau_I \tau}{K_c K_p}\right)s^2 + \left(\frac{1 + K_c K_p}{K_c K_p}\right)\tau_I s + 1 \]

Note:

- offset is removed
- closed-loop is second order
Example (continued)
effect of integral time constant and controller gain on closed-loop dynamics

natural period of oscillation

\[ \tau_{cl} = \frac{\tau_I \tau}{K_c K_p} \]

damping coefficient

\[ \xi = \frac{1}{2} \frac{\sqrt{K_p \tau}}{\tau} \frac{1}{\sqrt{K_c \tau_I}} \left( \frac{K_c K_p + 1}{K_c K_p} \right) \]

integral time constant and controller gain can induce oscillation and change the period of oscillation
PI Feedback

Effect of integral time constant on servo dynamics

\[ K_c = 1 \]
PI Feedback

Effect of controller gain

- affects speed of response
- increasing gain eliminates offset quicker
PI Feedback

Effect of integral action of regulatory response

- reducing integral time constant removes effect of disturbances
- makes behavior more oscillatory
PI Feedback

Important points:

- Integral action increases order of the system in closed-loop

- PI controller has two tuning parameters that can independently affect
  - Speed of response
  - Final response (offset)

- Integral action eliminates offset

- Integral action
  - Should be small compared to proportional action
  - Tuned to slowly eliminate offset
  - Can increase or cause oscillation
  - Can be de-stabilizing
Derivative Control

The function of derivative control action is to anticipate the future behavior of the error signal by considering its rate of change.

- The anticipatory strategy used by the experienced operator can be incorporated in automatic controllers by making the controller output proportional to the rate of change of the error signal or the controlled variable.
• Thus, for *ideal* derivative action,

\[ p(t) = \bar{p} + \tau_D \frac{de(t)}{dt} \quad (8-10) \]

where \( \tau_D \), the derivative time, has units of time.

For example, an ideal PD controller has the transfer function:

\[ \frac{P'(s)}{E(s)} = K_c \left( 1 + \tau_D s \right) \quad (8-11) \]

• By providing anticipatory control action, the derivative mode tends to stabilize the controlled process.

• Unfortunately, the ideal proportional-derivative control algorithm in Eq. 8-10 is *physically unrealizable* because it cannot be implemented exactly.
• For analog controllers, the transfer function in (8-11) can be approximated by

\[
\frac{P'(s)}{E(s)} = K_c \left( 1 + \frac{\tau_D s}{\alpha \tau_D s + 1} \right) \tag{8-12}
\]

where the constant \( \alpha \) typically has a value between 0.05 and 0.2, with 0.1 being a common choice.

• In Eq. 8-12 the derivative term includes a derivative mode filter (also called a derivative filter) that reduces the sensitivity of the control calculations to high-frequency noise in the measurement.
Derivative Action

Important Points:

- Characteristic polynomial is similar to PI
- derivative action does not increase the order of the system
- adding derivative action affects the period of oscillation of the process
  - good for disturbance rejection
  - poor for tracking
- the PID controller has three tuning parameters and can independently affect,
  - speed of response
  - final response (offset)
  - servo and regulatory response
- derivative action
  - should be small compared to integral action
  - has a stabilizing influence
  - difficult to use for noisy signals
  - usually modified in practical implementation
Proportional-Integral-Derivative (PID) Control

Combination of the proportional, integral, and derivative control modes as a PID controller.

- Many variations of PID control are used in practice.
- Three most common forms.

Parallel Form of PID Control

The parallel form of the PID control algorithm (without a derivative filter) is given by

\[ p(t) = \bar{p} + K_c \left[ e(t) + \frac{1}{\tau_I} \int_0^t e(t^*) \, dt^* + \tau_D \frac{de(t)}{dt} \right] \quad (8-13) \]

The corresponding transfer function is:

\[ \frac{P'(s)}{E(s)} = K_c \left[ 1 + \frac{1}{\tau_I s} + \tau_D s \right] \quad (8-14) \]
PID Feedback

Transfer Function

\[ U'(s) = K_c \left( 1 + \frac{1}{\tau_I s} + \tau_D s \right) E(s) \]

Closed-loop Transfer Function

\[ Y(s) = \frac{G_p(s)G_v(s)K_c \left( \frac{\tau_D \tau_I s^2 + \tau_I s + 1}{\tau_I s} \right)}{1 + G_p(s)G_v(s)K_c \left( \frac{\tau_D \tau_I s^2 + \tau_I s + 1}{\tau_I s} \right) G_m(s)} R(s) + \]

\[ \frac{1}{1 + G_p(s)G_v(s)K_c \left( \frac{\tau_D \tau_I s^2 + \tau_I s + 1}{\tau_I s} \right) G_m(s)} D(s) \]

\[ \uparrow \text{ Slightly more complicated than PI form} \]
PID Feedback

Example:

PID Control of a first order process

\[ G_p(s) = \frac{K_p}{\tau s + 1}, \quad G_v(s) = 1, \quad G_m(s) = 1 \]

Closed-loop transfer function

\[
Y(s) = \frac{\tau_D \tau_I s^2 + \tau_I s + 1}{\left( \frac{\tau_I \tau}{K_c K_p} + \tau_D \tau_I \right)s^2 + \left( \frac{1 + K_c K_p}{K_c K_p} \right) \tau_I s + 1} R(s) + \\
\left( \frac{\tau_I \tau}{K_c K_p} \right)s^2 + \left( \frac{\tau_I}{K_c K_p} \right)s D(s)
\]
PID Feedback

Effect of derivative action on servo dynamics
PID Feedback

Effect of derivative action on regulatory response

- increasing derivative action reduces impact of disturbances on control variable
- slows down servo response and affects oscillation of process
Tank with PI and PID Control

Proportional-integral: \[ G_c(s) = K_c \left\{ 1 + \frac{1}{T_I s} \right\} \]

\[ G_{servo} = \frac{K_c (T_I s + 1)}{T_I s (\tau s + 1) + K_c (T_I s + 1)} = \frac{K_c (T_I s + 1)}{T_I \tau s^2 + T_I (1 + K_c) s + K_c} \]

Proportional-integral-derivative:

\[ G_c(s) = K_c \left\{ 1 + \frac{1}{T_I s} + T_d s \right\} \]

\[ G_{servo} = \frac{K_c (T_I T_d s^2 + T_I s + 1)}{(T_I \tau + K_c T_I T_d) s^2 + T_I (1 + K_c) s + K_c} \]
Comparison between PI and PID
Reverse or Direct Action

• The controller gain can be made either negative or positive.

• For proportional control, when $K_c > 0$, the controller output $p(t)$ increases as its input signal $y_m(t)$ decreases, as can be seen by combining Eqs. 8-2 and 8-1:

$$p(t) - \bar{p} = K_c \left[ y_{sp}(t) - y_m(t) \right]$$  \hspace{1cm} (8-22)

• This controller is an example of a reverse-acting controller.

• When $K_c < 0$, the controller is said to be direct acting because the controller output increases as the input increases.

Note: This definition is based on input signal, $y_m(t)$ rather than the error signal, $e(t)$
Figure 8.11 Reverse and direct-acting proportional controllers. (a) reverse acting \( (K_c > 0) \). (b) direct acting \( (K_c < 0) \)
• **Example:** Flow Control Loop

Assume FT is direct-acting.

Should the flow controller have direct or reverse action?

1. Air-to-open (fail close) valve $\Rightarrow$ ? Reverse
2. Air-to-close (fail open) valve $\Rightarrow$ ? Direct
Example: Liquid Level Control

- Control valves are air-to-open
- Level transmitters are direct acting

Questions: Type of controller action?
Controller Comparison

P
- Simplest controller to tune ($K_c$).
- Offset with sustained disturbance or setpoint change.

PI
- More complicated to tune ($K_c, \tau_i$).
- Better performance than P.
- No offset, but tend to make the response more oscillatory.
- Most popular FB controller.

PID
- Most complicated to tune ($K_c, \tau_i, \tau_D$).
- Better performance than PI.
- No offset.
- Derivative action may be affected by noise.
Automatic and Manual Control Modes

- **Automatic Mode**
  Controller output, \( p(t) \), depends on \( e(t) \), controller constants, and type of controller used.
  (PI vs. PID etc.)

- **Manual Mode**
  Controller output, \( p(t) \), is adjusted manually.

- **Manual Mode** is very useful when unusual conditions exist:
  - plant start-up
  - plant shut-down
  - Emergencies

- **Percentage of controllers "on manual"** ??
  (30% in 2001, Honeywell survey)
**On-Off Controllers**

- The controller output of ideal on-off controller.

\[
u_{on-off}(t) = \begin{cases} u_{max} \\ u_{min} \end{cases}
\]

where \( u_{max} \) and \( u_{min} \) denote the on and off values, respectively.

- On-off controller can be considered to be a special case of P controller with a very high controller gain.

- Advantage: Simple and inexpensive controllers.
- Disadvantage
  - Not versatile and ineffective.
  - Continuous cycling of the controlled variable and excess wear on the final control element. And thus, limited use in process control.
- Usage: Thermostats in heating system.
  - Domestic refrigerator.
  - Noncritical industrial applications.
8.3.4 Typical Response of Feedback Control Systems

Figure 8.7. Typical process response with feedback control.

- **No feedback control** make the process slowly reach a new steady-state.
- **Proportional control** speeds up the process response and reduces the offset.
- **Integral control** eliminates offset but tends to make the response oscillatory.
- **Derivative control** reduces both the degree of oscillation and response time.

C is the deviation from the initial steady-state.
8.3.4.1 Effect of controller gain \( k_c \)

- Increasing the controller gain.
  - less sluggish process response.
- Too large controller gain.
  - undesirable degree of oscillation or even unstable response.
- An intermediate value of the controller gain
  - best control result.

Figure 8.8. Process response with proportional control.
8.3.4.2 Effect of integral time

Figure 8.9. PI control: (a) effect of integral time (b) effect of controller gain.

- Increasing the integral time.
  - more conservative (sluggish) process response.
- Too large integral time.
  - too long time to reach to the set point after load upset or set-point change occurs.
- Theoretically, offset will be eliminated for all values of $\tau_I$. 

\[ I \tau \]

Increasing $\tau_I$
8.3.4.3 Effect of derivative time \( \tau_D \)

![Diagram showing the effect of derivative time on PID control response]

Figure 8.10. PID control: effect of derivative time.

- Increasing the derivative time.
  - Improved response by reducing the maximum deviation, response time and the degree of oscillation.
- Too large derivative time.
  - Measurement noise tends to be amplified and the response may be oscillatory.
- Intermediate value of \( \tau_D \) is desirable.